Putnam Practice Problems Set #7

**Problem 1:** Let $A_1, A_2, \ldots, A_{1066}$ be finite subsets of a finite set $X$ such that $|A_i| \geq \frac{1}{2} |X|$ for every $1 \leq i \leq 1066$. Prove there exists ten elements $x_1, \ldots, x_{10} \in X$ such that every $A_i$ contains at least one of $x_1, \ldots, x_{10}$.

**Problem 2:** Let $I_m = \int_0^{2\pi} \cos(x)\cos(2x) \cdots \cos(mx) dx$. For which integers $m$, with $1 \leq m \leq 10$, is $I_m \neq 0$?

**Problem 3:** Let $f_0(x) = e^x$ and $f_{n+1}(x) = xf_n'(x)$ for $n = 0, 1, 2, \ldots$. Show that

$$
\sum_{n=0}^{\infty} \frac{f_n(1)}{n!} = e^e.
$$

**Problem 4:** How many primes among the positive integers, written as usual in base 10, are such that their digits are alternating 1’s and 0’s, beginning and ending with 1?