Problem 1: Evaluate \[ \int_0^a \int_0^b e^{\max\{b^2x^2, a^2y^2\}} \, dy \, dx, \] where \( a \) and \( b \) are positive.

Problem 2: Evaluate \[ \int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}}. \]

Problem 3: Evaluate \[ \int_0^{2\pi} \frac{dx}{1 + e^{\sin(x)}}. \]

Problem 4: Let \( p(x) = 2 + 4x + 3x^2 + 5x^3 + 3x^4 + 4x^5 + 2x^6 \). For \( k \) with \( 0 < k < 5 \), define
\[ I_k = \int_0^\infty \frac{x^k}{p(x)} \, dx. \]
For which \( k \) is \( I_k \) smallest?

Problem 5: For each continuous function \( f : [0,1] \to \mathbb{R} \), let
\[ I(f) := \int_0^1 x^2 f(x) \, dx \quad \text{and} \quad J(f) := \int_0^1 x(f(x))^2 \, dx. \]
Find the maximum value of \( I(f) - J(f) \) over all such functions \( f \).