Putnam Practice Problems Set #8

Problem 1: Suppose that we have integers $x, y, z$ with $z > 1$. Show that
\[(x + 1)^2 + (x + 2)^2 + \cdots + (x + 99)^2 \neq y^2.\]

Problem 2: A sequence of real numbers is defined recursively as follows: $x_0$ and $x_1$ are arbitrarily chosen positive real numbers, and
\[x_{n+2} = \frac{1 + x_{n+1}}{x_n} \quad \text{for} \quad n = 0, 1, 2, \ldots.\]
Find $x_{2007}$.

Problem 3: Around a circle are placed 20 ones and 30 twos so that no three consecutively placed numbers are equal. Find the sum of the product of every three consecutively placed numbers.

Problem 4: Prove that it is impossible to load a pair of dice so that every sum 2, 3, \ldots, 12 is equally likely. Assume that the dice are distinguishable, that is, a 2 on the first die and a 4 on the second die is different from a 4 on the first die and a 2 on the second, even though the same total of 6 is obtained.