Problem 1: Given 10 points inside a circle of diameter 5, prove that some distance between a pair of these points is no greater than 2.

Problem 2: Given 61 points inside a circle of radius 4, prove that some distance between a pair of these points is no greater than $\sqrt{2}$.

Problem 3: Prove that, for every set $X = \{x_1, x_2, \ldots, x_n\}$ of real numbers, there exists a non-empty subset $S$ of $X$ and an integer $m$ such that

$$\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}.$$