Problem 1: Suppose $p(x)$ is a polynomial of degree seven such that $(x - 1)^4$ is a factor of $p(x) + 1$ and $(x + 1)^4$ is a factor of $p(x) - 1$. Find $p(x)$.

Problem 2: Suppose that $a_0 = a_1 = 1$ and $a_n = 6a_{n-1} - 9a_{n-2} - 2^n$ for $n \geq 2$. Find $a_{2008}$, and your answer may contain numbers in the form $c(k^n)$.

Problem 3: Suppose $f$ is continuous in $[a, b]$, differentiable on $(a, b)$, and $f(a) = f(b) = 0$. Further assume that $r > 0$ is given. Show there exist a number $\xi \in (a, b)$ such that

$$rf' (\xi) + f(\xi) = 0.$$
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Solution:
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