Math 1190 Quiz #12

**Problem 1:** Prove that from a set of 10 distinct two-digit numbers, it is possible to select two disjoint subsets whose members have the same sum.

**Problem 2:** Let \( x_1, x_2, x_3, \ldots \) be a sequence of nonzero real numbers satisfying
\[
x_n = \frac{x_{n-2}x_{n-1}}{2x_{n-2} - x_{n-1}} \quad \text{for } n = 3, 4, 5, \ldots.
\]
Establish necessary and sufficient conditions on \( x_1 \) and \( x_2 \) for \( x_n \) to be an integer for infinitely many values of \( n \).

**Problem 3:** For any positive integer \( n \), let \( \langle n \rangle \) denote the closest integer to \( \sqrt{n} \). Evaluate
\[
\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} - 2^{-\langle n \rangle}}{2^n}.
\]
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