Math 1190 Quiz #3

Problem 1: Let v be a vertex of a cube C with edges of length 4. Let S be the largest solid sphere that can be inscribed in C. Let R be the region consisting of all points p between S and C such that p is closer to v than to any other vertex of the cube. Find the volume of R.

Problem 2: Inscribe a regular pentagon inside a unit circle and let A_0, A_1, A_2, A_3, A_4 be the vertices of the pentagon, in clockwise order. Show that

$$(|A_0A_1| \; |A_0A_2|)^2 = 5,$$

where $|A_iA_j|$ denotes the length of the chord which connects A_i and A_j.
(Hint: It may be worth noting here that if you were to inscribe a regular, 10-sided polygon in a circle of radius 1, then it would have side length $\frac{1}{2}(\sqrt{5}-1)$.)