

Review of Athreya & Lahiri, *Measure Theory and Probability Theory*

Probabilists have a special relationship to measure theory. Whereas mathematicians may often view measure theory mostly through its applications to Lebesgue measure on Euclidean spaces, probabilists routinely also deal with spaces of sequences, trees, functions, and other objects where the relevant measures can be understood only from a solid foundation in general measure theory. This divide is reflected in how measure theory is being taught to graduate students; those focusing on mathematics usually learn measure theory from courses in real analysis and may use, for example, texts by Folland or Royden (both titled *Real Analysis*). On the other hand, students who focus on probability and statistics usually learn their measure theory from courses in advanced probability theory where real analysis plays an important, yet smaller, role, the stress being instead on the construction of probability measures, expectation as an integral, and conditional expectation as a Radon-Nikodym derivative. Some standard texts are Chung, *A Course in Probability Theory*; Billingsley, *Probability and Measure*; and Resnick, *A Probability Path*.

Anybody who has taught a course in measure-based probability has faced the problem of how to start. Should one start with probability measures and develop the general theory later or should one start with general measures and introduce probability as a special case? Billingsley is an example of somebody who favors the first approach and, although in many ways a very good text, it becomes a bit clunky when many of the results and arguments for probability measures are repeated and extended for general measures. In *A Modern Approach to Probability Theory* (Birkhäuser, 1996) Fristedt and Gray take the same route but hold off with proofs of the central theorems until the sections on general measure. Other advocates of the first approach are Chung, who in the third edition of his classic text puts measure theory in an appendix, and Resnick, who skips general measures altogether.

It is of course a matter of taste but I decidedly favor the second approach, to go from the general to the specific, provided that the two prime special cases – probabilities and Lebesgue measure – are brought in early as concrete examples of measures. This philosophy is implemented in Athreya's and Lahiri's recent addition to the literature. Almost encyclopedic in scope, the book provides not only extensive coverage of standard measure theory, real and functional analysis, and graduate level probability theory, but also

Markov chains, renewal theory, and Brownian motion, as well as more recent topics such as the Black-Scholes formula, Markov chain Monte Carlo methods, and bootstrapping. The competence of the authors is unquestionable. Both have impressive publication records, currently adding a combined total of about 150 items to MathSciNet, and both remain active researchers and are also experienced teachers. Their fields of expertise are represented in the book in sections on branching processes (Athreya) and bootstrapping (Lahiri).

The style of writing is clear and precise, combining the Eurasian tradition of stringency and attention to detail with the New World demand for relevance and applicability. The text progresses at a fairly rapid pace but the authors nevertheless strive to give intuitive explanations of the various concepts and also provide an ample supply of examples and problems.

Its wide range of topics and results makes *Measure Theory and Probability Theory* not only a splendid textbook but also a nice addition to any probabilist's reference library. I view the book from the point of view of a probabilist, but it should be pointed out that the first five chapters can be used for a pure mathematics course in measure theory and real analysis without any reference to probability. Whether you are an instructor who has not found a text to your liking or wishes to try something new, a researcher in need of a reference work, or just somebody who wants to learn some measure theory to lighten up your life, *Measure Theory and Probability Theory* is an excellent text that I highly recommend.

Peter Olofsson  
Trinity University