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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Classical Mechanics. *By R. Douglas Gregory*. Cambridge University Press, Cambridge, UK, 2006. $120.00. xii+596 pp., hardcover. ISBN 978-0-521-53409-3.

Writing a book on classical mechanics is a very daring enterprise. Hundreds of people have done it before you, so you must be ready to face critical remarks from different perspectives. Be sure that roughly half of your readers will say that there is not enough physics in the book, and the other half will complain about a lack of mathematical rigor. I belong to the second half, but let's start with the good news: this is one of the better books, in many respects; it deserves a place in every library which covers the subject and on the list of recommended reading accompanying an undergraduate mechanics course.

In his preface, the author says that "a previous course in mechanics is helpful but not essential." That struck me as interesting, because I personally have found myself for years in a similar position: teaching theoretical mechanics both to students who have had such a preliminary course and others who haven't. The preliminary course is offered to physics students and is typically taken from volume 1 of some series on general physics. In my opinion, there are many excellent books one can take as a guideline for such a purpose. The situation is different, however, when it comes to finding a good book for a first course on theoretical mechanics, which is supposed to start from scratch, but should cover the subject in more generality, should pay particular attention to the fundamentals of the theory, and bring it to a point from which other courses on mathematical physics can take off. I must admit I have never found one that gives me complete satisfaction, and there must be other people, like Douglas Gregory perhaps, who have had the same feeling and come to the conclusion, "Why don't I write my own?"

If you do teach an undergraduate course on theoretical mechanics to mathematics and physics students, which nowadays is becoming a privilege, you better take the issues of basic principles and subsequent consistent mathematical developments seriously, because there will be no opportunity to come back to it in greater depth in a later course. Far from wanting to insinuate that Gregory would not take this issue seriously, my feeling is that the book has a number of shortcomings in that respect. I was put somewhat on alert (a warning of bigger disasters to come later) with Definition 1.1 on the first page in the book, which says, "If a quantity Q has a magnitude and a direction associated with it, then Q is said to be a vector quantity." If that is supposed to be a mathematical definition, I can only raise the question, "What on earth is a quantity?" And it is a pity because, if you turn the page, you find a nice figure showing different representations of vectors forming a parallelepiped, which has all the ingredients for a proper definition of the concept of a vector. Putting that aside, the first two chapters give a good buildup of the prerequisites concerning vector

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calculus and kinematics, except for the following remark. When we get to the discussion of reference frames in relative motion, only translational motion is considered, which after all is not very common in practice. We have to wait until Chapter 17, long after we have been told about Lagrangian and Hamiltonian systems, before the important concept of the angular velocity vector is introduced.

Then comes Chapter 3, which is of course about Newton's laws, and which remains the most delicate part of such a book. I believe that more than 300 years after the master, nobody can give a completely indisputable account of the basic principles of Newtonian mechanics, in the sense of setting up a mathematical model, and Newton's laws should somehow be the axioms from which everything else in the model can be deduced. Many books can be caught on real inconsistencies and circular arguments in this respect. The present author is aware of this danger and generally does a good job. But having discussed the axiom about existence of inertial frames and the introduction of the concept of mass, Gregory chooses the easy way out by considering force as being *defined* as mass times acceleration. I have a problem with such an interpretation, because it is somehow contrary to the whole purpose of setting up a mathematical model. It condemns us to being useless spectators in the whole theory: all we can do is observe motions, measure mass and acceleration at each instance of time, and then conclude that the product of the two is what we have defined to be the force at that time! The whole purpose of setting up a model is of course to be able to make predictions, rather than simply observing. In addition, our intuition is that force is an independent concept. Having made this point, I feel obliged to offer an alternative, knowing that everything I say now can be used against me. I postulate the principle in question roughly as follows: Every nonisolated particle with mass m moves, with respect to an inertial frame, in such a way that $\forall t$, we have $m\mathbf{a}(t) = \mathbf{F}(t)$, where $\mathbf{F}(t)$ represents a *physically interpretable force*. A force being physically interpretable then essentially means that, from experimental observations where one measures $m\mathbf{a}(t)$ for different values of time (in reproducible experiments), one has to discover a "force law," i.e., a vector function $\mathbf{F}(\mathbf{x},\mathbf{v},t)$ such that at all times during the observed motion, the measured $\mathbf{F}(t)$ is equal to $\mathbf{F}(\mathbf{x}(\mathrm{t}),\mathbf{v}(t),t)$. This is the step which makes the model useful: from then on, we become intelligent creatures who can predict phenomena by solving differential equations. Moreover, the fact of discovering a force law serves at the same time as justification for the conclusion that the reference frame which was used in the process is a sufficiently good approximation of an inertial frame (at least for the kind of interaction under consideration). There is an interesting consequence of such an approach! (I am beginning to realize that writing a review of a book on classical mechanics is an equally daring enterprise, because I am about to overthrow a sacred principle!) At some point, say, based on the experimental observations which led to Kepler's laws, our fantastic gravitational force law, with magnitude mMG/r^2, is discovered. Later on, for certain phenomena believed to be attributed to gravitation, more precise measurements might indicate that the results are not quite in agreement with the theoretical predictions. Then there is a variety of possible explanations: (i) maybe the assumption that we had a good approximation of an inertial observer should be improved (which is what we do when discussing the effect of the rotation of the earth on falling objects); (ii) the gravitational constant G in the model is perhaps not quite a constant and we should think of something better; (iii) even more dramatic changes to Newton's gravitational model should be considered; or there is another mysterious force in action, etc. But there is absolutely no logic in arguing from the beginning that the mass factors mM

in Newton's gravitational law should relate to something different from the concept of mass introduced in the first few axioms, then setting up all sorts of complicated experiments to test this (in an earthly environment even!), only to conclude that, miraculously(?), inertial and gravitational mass are the same. In other words, the principle of equivalence is a complete artifact (in classical mechanics).

In Chapter 4, we find a good collection of examples of solving relatively simple problems. The exercises look very good and challenging. Chapter 5 offers the classical treatment of simple, damped, and driven oscillations, but includes a few more advanced topics as well, such as the use of Fourier series and coupled oscillations and normal modes (in two dimensions). I miss a couple of features in the qualitative discussion of one-dimensional conservative motion in Chapter 6, for example, the fact that an unstable equilibrium can be approached only in an infinite time. Also, for three-dimensional motion, there is no mention of the test conditions for a force law to be conservative. Chapter 7 is devoted to central forces and thus is important. It is rich in content, including, for example, a nice discussion of Hohmann transfer orbits and Rutherford scattering. I also appreciate that elements of perturbation analysis are used throughout the book, when it is suitable. Concerning the discussion of the paradigm central force problem, which is of course Kepler's problem, there is a nice derivation of Kepler's equation for determining the position of the planet on its orbit, for example. Yet, it is a pity that there is no mention of the Laplace–Runge–Lenz vector constant (which is understandable though, in the author's conception, because even the angular momentum vector is only introduced much later). Chapter 8 is an interesting digression on nonlinear effects. It may look a bit strange that there is so much focus on phase space analysis in two dimensions (with the Poincaré–Bendixson theorem, for example). But the justification is that it makes the link with the linear oscillations of Chapter 5 and with conservative one-dimensional motion (though this link should have been made more explicit by also discussing the phase portrait, as derived from the study of the graph of the potential energy function).

Part 2 of the book carries the title "Multi-particle Systems and Conservation Principles." It is a nice idea to organize this material in three chapters, in which consecutively the emphasis is on the impact of conservation of energy, conservation of linear momentum, and conservation of angular momentum. It is unfortunate, however, that the author calls these conservation theorems or the equations from which they derive "principles." The term principle should be reserved for the fundamental laws, or axioms, or assumptions of the theory (whatever you want to call them). For example, the author talks about d'Alembert's principle later on, which is fine because this concerns an additional assumption which is thrown in at some stage and which is valid for all systems...for which it is not violated. This brings us back to the issue of fundamentals, and I must say that there is again an alarm signal at the very beginning of Part 2. In the description of "Key Features" of Chapter 9, we read, "In particular, multi-particle mechanics is needed to solve problems involving the rotations of rigid bodies." In the author's defense, he defines rigid bodies at first as rigid arrays of particles, which is fine. But that is not how rigid bodies present themselves in applications, and to tacitly make the transition from a finite array of particles to a continuous mass distribution means sweeping a couple of conceptual difficulties under the carpet. Having proved that gravity acts on a system of particles as though the total mass is concentrated in the center of mass, and then taking this for granted when also applied to something like a uniform plank (p. 233), is still acceptable, because a standard limit process can make the transition from a finite sum to an integral rigorous in this case. But things are different regarding specific forces (for example, springs attached to specific points of the body), which are not force fields defined by a certain force density. Then, to prove for a system of particles the well-known theorems about the motion of the center of mass and the rate of change of angular momentum, and to proclaim these to be the (proved) rigid body equations (Chapter 11), is rather misleading, in my opinion. It is not so easy to get around this difficulty: my feeling is that we don't want to start talking in such a course about distributions (force fields which have a kind of δ-function density, roughly speaking). My preferred way out, therefore, is to introduce the above-mentioned equations, called Euler's laws, as the *axioms* for rigid body motion, on the same level as Newton's laws for particles, and of course duly motivated by the fact that they become theorems in the case of a rigid array of particles. I further regret that we are still not taught about the nature of general rigid body motion in this part. True, there is a paragraph which carries the title "Rigid body in general motion" in Chapter 9, but it unfortunately adds to the haziness about the subject. Indeed, the author discusses the splitting of the total kinetic energy into the kinetic energy of the center of mass and a remainder term, which he then claims to have the "nice physical interpretation" as having the form $\frac{1}{2}I\omega^2$. But a general motion of a rigid body will of course not be a (translational) motion of its center of mass G plus a rotation about a fixed axis through G superimposed on this. By the way, there is a reference in this chapter to an appendix where moments of inertia are computed. They are defined as a finite sum again, but then computed via integrals (as something which is self-evident) for the typical examples of bodies with symmetry and a constant mass density. In between all these remarks about basic principles, I should not forget to say that this part of the book again contains a large number of interesting topics, among which I mention the two-body problem, collision and scattering problems, the use of the center of mass frame in general, the spherical pendulum, and rigid body static problems.

Part 3 also has three chapters and is called "Analytical Mechanics." The first chapter introduces generalized coordinates, d'Alembert's principle and Lagrange's equations (for systems of particles) as derived from it, and finally Noether's theorem, or at least a reduced version of it. The latter is a bit strange, since Noether's theorem can be dealt with so much better in the context of the calculus of variations, which is the subject of the next chapter. The treatment of the calculus of variations and Hamilton's principle is quite acceptable, though I don't see the need for limiting the analysis first to one-degree-of-freedom systems and then repeating the arguments for many degrees of freedom. As before, there are interesting and challenging exercises, almost always originating from realistic sounding problems. Concerning the transition to Hamilton's equations in Chapter 14, I again don't see a compelling need for discussing the Legendre transform first for the two-variable case. I further abstain from comments about the author's usage of "imagined substitutions," something I have never seen before. But it seems to me that a simplified account of the inverse and implicit function theorem would be welcome here. A point of merit in this chapter is the discussion of Liouville's theorem and recurrence. Incidentally, I spotted an obvious factor 2 mistake in Exercise 14.9, which proposes a variational principle for Hamilton's equations, but there is a slightly more annoying factor 2 problem in the next chapter on the theory of small oscillations.

Chapter 15 is actually the first one of Part 4 on "Further Topics." It has all the usual contents about small oscillations and normal modes, treated within a Lagrangian setup. The author strangely omits the factor $\frac{1}{2}$ in the quadratic term of the Taylor expansion of the potential energy function. I have a sneaking suspicion that this is the reason why he chose earlier to represent the quadratic term of the kinetic energy in generalized coordinates without a factor $\frac{1}{2}$ as well, which is permitted but rather unusual. This has the effect of making things fall in the right place in this chapter. Another remark is that the proof about a sufficient condition for stability of an equilibrium point, in my opinion, requires this equilibrium to be a *strict* minimum of the potential energy, and then the strange extra assumption that it should be a minimum of the approximate potential goes away as well. Chapters 16 and 17 are actually very good: at last we are told about general rigid body kinematics and general features of relative motion and particle dynamics with respect to noninertial frames. In my opinion, however, these are not issues one can classify under "further topics"; they really should come at the very beginning of the book. The author's opinion becomes clear when he says, "We now prove the fundamental theorems of rigid body kinematics. The details of the proofs are mainly of interest to mathematics students." With all due respect, I totally disagree with this point of view: if we are talking about "fundamental theorems" (and indeed we are!), then they must be of interest to physics students as well!

Chapter 18: the dreaded disaster strikes! This chapter is about tensor algebra and the inertia tensor. The author warns that "Any account of tensor algebra is mathematics, not mechanics, and some readers may find this indigestible." He is right but for the wrong reasons: it is the pseudomathematics which makes it intolerable. I know of course that in numerous physics books, tensors are defined essentially as things, with little things attached to them (called indices), which respond in a certain way when we change coordinates. Well, we can have a little fun with this, but it is usually not all that dramatic if one quickly passes to the calculations with tensors and doesn't start asking silly questions like, "What on earth is a quantity?" But here the text is a bit harder to swallow, because the definitions start with mathematical nonsense of the following kind: "Let ϕ be a real number defined in each coordinate system ..." (for a scalar), or "Let $\{v_1, v_2, v_3\}$ be a set of three real numbers defined in each coordinate system ..." (for a vector). Anyhow, the point I would really like to make (again) in this context is the following: physics students should not be scared away from a little bit of elementary mathematics. Aren't they supposed to get a basic course on linear algebra, for example? Well then, what's wrong about talking about a real vector space V (the term is never used in this book) and its dual V^*: the space of linear maps from V to $\mathbb{R}$? Then a tensor becomes a multilinear map from a number of copies of V and V^* to $\mathbb{R}$, etc. What physics students are supposed to understand later on, when they study more advanced theories such as quantum mechanics, relativity theory, string theory, etc., often involves a level of abstract thinking which is much more abstract than any abstract mathematical subject one can think of! Coming back to the issue under consideration here, I agree to some extent with the author that one can deal with rigid body dynamics without knowledge of tensors. This is all the more true, for example, if all one says about the inertia tensor (as many books do) is that it is a certain matrix or, a little bit better, a linear map which transforms the angular velocity vector into the angular momentum vector. Then why use the term tensor? The main reason for talking about an inertia tensor (rather than an inertia matrix) is in fact its appearance as a bilinear form in the kinetic energy: it is the metric tensor of rigid body motion.

Let us pass to the final Chapter 19, which treats problems in rigid body dynamics. As I said before, what I called Euler's laws are taken for granted as starting equations here, even though they were proved to hold only for systems of particles. Not surprisingly (in fact this was already the case in Chapter 12), Lagrange's equations are also taken for granted as a model for rigid body motion, even though their validity was proved to follow from Newton's principles and d'Alembert's principle only for systems of particles. Putting these remarks aside, the selection of topics and their treatment in this chapter is extremely well done, apart from one slip of the tongue: as an illustration of the "deficiency of Euler's (dynamical) equations," the author claims that the right-hand sides of these equations, in the case of the spinning top, are unknown! This is not true, as it is easy to express the vertical unit vector (needed for the computation of the moment of the gravitational force) in terms of the body frame, using Euler's angles. It is to be regretted also that there is no mention at all of Euler's kinematical equations, especially in the context of the last question the author addresses: if the $\omega_i(t)$ are known, what is the corresponding motion of the body?

In summary, my list of comments may create the wrong impression that Gregory's book is very bad, but that is not the case: I did mean what I said at the beginning. The point is that a number of my critical remarks are not specific to this book, but apply to many books on classical mechanics. As such, they may well be rather controversial, in which case I would hope they stimulate further critical examination of basic principles. If you are not so sensitive about fundamentals of the theory, and about a minimal care for a good mathematical education of physics students, then you will not be faced with stumbling blocks and can find lots of interesting topics and ideas in this book. Besides, the style of writing is very good, so it certainly makes pleasant reading. As for myself, I have still not found the book I want to adopt for my courses.

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Embedded Robotics: Mobile Robot Design and Applications with Embedded Systems. Second Edition. *By Thomas Bräunl.* Springer-Verlag, Berlin, 2006. \$59.95. xiv+458 pp., softcover. ISBN 978-3-54034-318-9.

Robotics is continuously evolving, moving from the industrial to the everyday-life environment. Robots are progressively invading human spaces; as an example, today a large number of robotic vacuum cleaners are in use in private habitations. The robotics community is spending significant effort toward making robotic devices coexist with humans. Following this trend, in recent years several universities have started offering classes that give the background for autonomous, embedded robotics.

This book provides an interesting overview on embedded robotics. The author imparts his experience acquired during several years of robot laboratory courses and from the results of an international project, named EyeBot, that involved institutions from Australia, Canada, Germany, New Zealand, and the U.S.

One of the achievements of the project was the development of an embedded controller, named EyeCon, developed with a customized operating system, named RoBIOS. Starting from this controller, during the research and the teaching activities, a number of customized as well as commercial sensors and actuators were used on the design and implementation of several autonomous robots. The overall result is a modular robotic system used for mobile as well as walking or underwater robots.

The book presents most of the practical aspects related to the design and control of an autonomous robot. In the first part of the book the reader gains a good overview of the considerations to be made when selecting the sensors and actuators, while in the second part several robotic mechanics and applications are given. The last part refers to higher level aspects of autonomous robotics such as localization and navigation, and exploration with advanced concepts such as neural networks or genetic programming. It is appreciated that the author gives the book a practical slant by reporting technicalities useful for practical implementations, thus avoiding the use of heavy formalisms. Moreover, the correct emphasis is placed on the importance of hardware in the loop simulations as part of the robotic design.

In the reviewer's opinion, this book is suitable as a textbook for a laboratory class on robotics. A student will learn most of the capabilities required to select the components and build, program, and control an autonomous robotic system. Since it is obviously not easy to produce a self-contained book for such a wide topic, and in order to really appreciate the book's contents, the reader should have a basic background in control/robotics.

The book might also be of interest to Ph.D. students when first reading about autonomous robotics; however, in the reviewer's opinion the book would not be of special interest to researchers with a mature background in this topic. In fact, in several chapters the bibliography does not seem to address the relevant citations; as an example, Chapter 12 contains six citations and four of them are theses; the Peters citation in Chapter 7 is in German. Furthermore, there is a certain nonhomogeneity in the depth of the various chapters' bibliographies, with some containing seminal books and journals and others reporting only conference papers.

Another comment concerns the relative weights among the topics: it is the reviewer's opinion that maybe classical control approaches are a little under-evaluated compared to genetic or neural network approaches.

In conclusion, the book is very well organized and it is written in a pleasantly concise style. Undergraduate and graduate students and researchers interested in embedded robotics will find it useful and rich in valuable material.

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A Java Library of Graph Algorithms and Optimization. *By Hang T. Lau*. Chapman & Hall/CRC, Boca Raton, FL, 2007. $99.95. x+386 pp., hardcover. ISBN 978-1-58488-718-8.

As the title suggests, this text describes a collection of routines in the Java programming language solving basic problems in graph theory, such as locating the connected components of a graph, and in graph optimization, such as finding a maximum network flow. Each of the 55 routines is preceded by an introduction about a page in length. This is split between a description of the problem to be solved, with some references to the literature, and a description of the parameters passed to the routine. This is followed by the verbatim listing of the code. The one-page conclusion of each routine's description is a small example, typically a graph on 8 or 10 vertices, and the program listing for a small driver program and its output.

So roughly 250 pages are given over to program listings. The use of the word "library" in the title would suggest some sort of uniformity to the routines and an interdependence, utilizing common data structures and exploiting the object-oriented features of Java for efficiency in use and presentation. This is not the case. There is not even a graph object. Typical data structures are arrays of integers, so graphs are implemented as paired one-dimensional arrays of endpoints of edges (`nodei[], nodej[]`) or as a two-dimensional array that is the adjacency matrix. Worse, the code takes no advantage of Java's strengths and is written in a style reminiscent of C, Pascal, or FORTRAN. Arguments to routines are often long lists of arrays, there are no objects in sight anywhere, and the output requires the user to interpret the contents of the arrays containing the results. Besides questions of style, the particular choice of algorithms also raises questions. The description of depth-first search ignores the possibility of a recursive approach. The long section on graph isomorphism makes no mention of McKay's algorithm (implemented as the nauty package and freely available for non-military use), and instead uses an algorithm from a FORTRAN90 library.

Of the roughly 100 pages that are not code, there is little explanation of the choice of algorithms or data structures for the problem at hand. A similar effort to this book is Skiena's *Implementing Discrete Mathematics* (Addison–Wesley, 1990), which describes the Combinatorica package for *Mathematica*. The reader who expects the work under review to match Skiena's careful attempts to teach and explain will be very disappointed. The book's introduction says, "The library of programs is intended to be used for educational and experimental purposes." While the use of such a library could be beneficial in an educational setting (or for rapid prototyping), the book itself makes very little effort to teach or inform.

Included inside the back cover is a copyrighted compact disc. There is no reference to this disc in the book, no description of its contents, no guidance on allowed uses. The closest thing to a mention of how the disc may be used is the standard boilerplate on the copyright page which says, "No part of this book may be utilized... in any information storage or retrieval system... without written permission from the publishers." Hazarding a copyright violation, an examination of the contents of the disc reveals it contains the class files of the routines. These are the intermediate, machine-readable files created by a Java compiler, and not the original source files as printed. So modifications to these files (perhaps for "experimental purposes") are not made easily. For a project of this nature, it would be more useful to release the source code in electronic form with a license explaining clearly what the purchaser is allowed to do with the code and with any programs that might incorporate it. A search uncovered no web locations that might also be hosting the source code. Using Skiena's work as an example again, consider that his *Algorithm Design Manual* (TELOS, 1998) is available online in a hypertext-linked edition and is supported by the comprehensive Stonybrook Algorithm Repository website.

There is little to recommend in this project. The text and the code are not instructive. Practical use of the class files on the compact disc for experimental purposes is limited at best, and prohibited as described in the front matter. Whether or not the algorithms are carefully chosen and implemented, whether or not the style of coding is appropriate or instructive, delivery as a printed book with an electronic supplement containing only machine-readable implementations seems ill-advised.

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Handbook of Mathematics for Engineers and Scientists. *By Andrei D. Polyanin and Alexander V. Manzhirov.* Chapman and Hall/CRC, Boca Raton, FL, 2007. $99.95. xxxii+1509 pp., hardcover. ISBN 978-1-58488-502-3.

How many hours are there in a day for Andrei Polyanin? Given his production over the last decade, simply measured in terms of reference books [3, 4, 5, 6, 7], the answer has to exceed 24. And now arrives the most massive of them all, the 1500-page handbook that is the subject of this review. The book is coauthored with Alexander Manzhirov, who was also a collaborator on [5]. That previous book has a far more limited scope: it focuses on a single topic, broad as it may be, and presents a seemingly exhaustive list of solutions to different integral equations. In addition, the user (I had written "reader," but corrected myself) finds sections on the methods leading to those formulae. The other reference texts [3, 4, 6, 7] are in the same vein. The current book is different. It is by far the most classical, and there are plenty of books available for direct comparison (see [1, 2, 8], for instance). Of these, CRC's own [8] is perhaps the best known. The most accurate comparison is probably with [1]. Both books not only list lots of facts, but also discuss methods, give theorems, and even include the occasional proof. For reference books such as this, one's favorite will often be the one first used, as this is the book that allows the user to find what is needed the fastest. Therefore I cannot recommend that we all toss our favorites, but I can safely recommend this volume to anyone who wants a reference.

What is to be found here? All the classical culprits are present (algebra, elementary functions, geometry, calculus, complex analysis, integral transforms, ordinary and partial differential equations, special functions, probability and statistics), as are some not-so-standard ones (integral equations, difference equations, calculus of variations). The book is split into two parts: methods (about 1100 pages) and tables (about 400 pages). Both parts are well structured, and well written. I have some quibbles with notation (often, the authors use y'_x to denote the derivative of $y(x)$; similarly y''_{xx} denotes the second derivative), but these are minor. Also, occasionally theorems are attributed to names that may not be canonical for the Western literature. For instance, Gauss's theorem is referred to as the Ostrogradsky–Gauss theorem. That may be neither here nor there, but it does affect how quickly one retrieves information using the index. An easy solution would be to have multiple index entries for these cases. On the plus side, the coverage of the topics included is excellent, often beyond what one would expect from an applied mathematics reference (Stieltjes integrals, inverse scattering, or game theory, anyone?). Due to the well-thought-out layout of the different chapters, the extra information provided does not seem to interfere with the effectiveness of getting to the desired information.

One may argue that some parts of the material included in the tables section appear somewhat arbitrary. After a first look I felt this second part of the book was a prolonged advertisement for the other reference books mentioned above [3, 4, 5, 6, 7], but that is unfair. What is somewhat arbitrary is where one draws the line on what to include in the table, and what to omit. I am not clear on how the authors reached their decision, and it is likely that the publisher had a say. Most important is that the elementary results one expects to find are present. Maybe there is a question of relevance for the tables section: Maple and *Mathematica* and other computer algebra systems are now so widespread that they are the first reference for elementary (and not so elementary) results for whole new generations of students. I think this criticism can be applied to several chapters of the tables section included here (integrals and ordinary differential equations), but it does not (yet) apply to most of the others: Maple and *Mathematica* still resort to table lookup themselves for integral transforms, and they offer limited assistance with such things as integral equations and symbolic solutions of partial differential equations.

For many of the results in the methods section, one may be tempted to go online to free sources such as wikipedia (http://www.wikipedia.org) or MathWorld (http://mathworld.wolfram.com), and one may wonder whether $99.95 is a wise investment for even such a fine reference text as this. Admittedly, this is something I am doing myself more and more. Maybe it is something Dr. Polyanin is doing as well these days: he is the main editor of EqWorld (http://eqworld.ipmnet.ru), another free resource. The contents of this text do not appear there, but there is serious overlap with [3, 4, 5, 6, 7]. Maybe there are plans to include the material of this text. This should be welcomed by the scientific community. If so, Dr. Polyanin may want to consult Eric Weisstein, who has some experience with this publisher. Until then, this is a fine reference text, offered at a very reasonable price in a good quality hardback.

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Applied Fuzzy Arithmetic: An Introduction with Engineering Applications. *By Michael Hanss*. Springer-Verlag, Berlin, 2005, $69.95. xiv+256 pp., hardcover. ISBN 978-3-54024-201-7.

Introduction. This book deals with the application of fuzzy set theory to mechanical engineering. More precisely, it focuses on an extension of interval methods where intervals are changed into fuzzy intervals. The thrust of this monograph is the presentation of a practical but general method for computing functions of fuzzy intervals. The main originality lies in the attempt to go beyond rational functions expressed by means of arithmetic operations, toward a systematic handling of general nonmonotonic functions. This is not the first book addressing the topic of fuzzy arithmetic, although there are not many of them. The first systematic treatment of this problem can be found in the 1980 book of Dubois and Prade [5], with developments in another book [8] (and with more details in [7]). The first dedicated monograph is that of Kaufmann and Gupta [13] from 1985. A more recent one is that of Mares [16]. An extensive account of more recent theoretical and practical developments along with an extensive bibliography is the survey paper by Dubois et al. [9]. The originality of *Applied Fuzzy Arithmetic* lies in its stress on computational methods which can be used beyond the four operations of arithmetic, and in the inclusion of reports on real applications. In this sense, this is an original and welcome addition to the literature.

Analysis of Contents. The content of the book can be described as follows. It is divided into two parts, one on the basics of fuzzy interval analysis and computational methods, and one devoted to application examples.

The first chapter recalls the necessary background on fuzzy set theory, especially the notion of a cut set of a fuzzy set, set-theoretic operations, basic facts on fuzzy relations, and the formal underpinnings of any computation with fuzzy intervals: the extension principle of Zadeh. This principle enables the image of fuzzy sets by a numerical function (whose arguments become fuzzy sets) to be defined. It is the fuzzy counterpart to the principle that prescribes how to carry a probability measure from one set to another via a function. In fact, the calculus of fuzzy intervals is a max-min–based counterpart to the calculus of random variables. On the other hand, the extension principle comes down to applying standard interval analysis to all cut sets of fuzzy intervals appearing as arguments of the function. This approach is useful in representing incomplete knowledge in scientific calculations and evaluating its impact on the knowledge of the results.

The second chapter deals with elementary fuzzy arithmetic. It surveys various shapes of fuzzy intervals and the general so-called LR-fuzzy numbers. These are specified by the choice of shape functions L and R, and differ from each other by a translation and a homothety. The shape of such fuzzy intervals is then invariant under addition. Closed forms for subtraction, division, and multiplication of LR-fuzzy numbers can be obtained [8]; however, the shape is no longer invariant for the two last operations, and L-R approximations of products and quotients are recalled in this chapter. They may lead to significant loss of information. In contrast, another approach is recalled, based on the use of cut sets. A fuzzy interval with a continuous membership function can be discretized by considering a finite set of membership grades (including 0 and 1). In fact the author considers so-called discretized fuzzy numbers characterized by a sequence of pairs each made of a real value and a membership grade. The extension principle is then applied directly to such discrete representations. However, this technique has many drawbacks as it is not always obvious how to preserve this representation through computations, nor how to retrieve the right membership grades. Hanss then considers so-called decomposed fuzzy numbers, understood as a finite nested family of cut sets. Fuzzy arithmetic applied to these decomposed fuzzy numbers comes down to interval arithmetic applied to the corresponding cut sets. Both discretized and decomposed fuzzy numbers yield the same results with arithmetic operations in theory, but using cuts of fuzzy sets is by far the most convenient and general representation. In fact, the difficulty with domain discretization of membership functions has been known since at least 1978 (see Dubois and Prade [4]).

The second chapter is devoted to exhibiting the limitations of standard fuzzy arithmetic as it is still used by a majority of authors. Such limitations were repeatedly pointed out in the past by several authors like Dubois and Prade [7], Dong and Wong [2], and Klir [14], among others. Namely, given a rational function involving fuzzy arguments, the fuzzy set image of the function cannot generally be obtained by substituting parameters by their fuzzy values and performing the four arithmetic operations in the order prescribed by the form of the rational function. In general, what is obtained is more imprecise than (i.e., a covering approximation of) the actual output fuzzy interval. Worse, different forms of the same rational function, which would yield the same result with precise numbers, yield different results with fuzzy sets when using fuzzy arithmetic, and sometimes none of them yields the exact result. Very simple examples of this phenomenon can be laid bare. For instance, substituting fuzzy intervals A and B for x and y in the expression $x+y-y$ is not equivalent to substituting A for x in the simplified expression without y. This is because $A - A$ is computed as the fuzzy range of $y - z$, where y and z are independent variables known to be restricted by the fuzzy set A.

One way of making sure of obtaining the most precise output is to express the function so that each fuzzy argument appears only once. But this is not always possible. When the same argument appears in several places, and independent variables substituted for them would not display the same monotonies (like $y-y$ becoming $y-z$), then fuzzy arithmetic cannot be applied. This phenomenon appears when computing a weighted average either as a rational fraction with the same weights appearing in the numerator and the denominator (a recent paper along this line being [11]), or as a convex sum, where the normalization constraint expresses a linear dependence between weights [6]. The book under review exhibits these limitations on a mechanical problem (computing the displacement of a cross section in a one-dimensional static problem involving a massless rod under a tensile load), as well as other simpler examples of polynomials and rational fractions.

Chapters 3 and 4 contain the meat of the book. They propose a general computational scheme, termed the transformation method, for computing functions of fuzzy variables without resorting to standard fuzzy arithmetic, which can deal with nonmonotonic functions. This approach considers a selection of m equidistant cuts of the fuzzy inputs and relies on computing precise values of the function using a suitably chosen sampling of input values in each considered cut. When the function is known not to contain extrema inside the supports of the fuzzy inputs, the function needs to be evaluated only on vertices of the hypercubes formed by each tuple of cuts of the fuzzy inputs. These vertices are tuples of endpoints of intervals generated by the cuts. This is the restricted transformation method.

When the monotony of the function is unknown, inner points of the hypercubes must be explored. The originality of the transformation method lies in the order in which the sample of evaluation points is generated. The technique starts with the cores of the fuzzy intervals that are singletons, and hence form a crisp point that will be the core of the result. Then it proceeds top down through decreasing values of membership thresholds, thus considering wider and wider cuts. Due to the nestedness of cuts, points computed at a certain membership level can be reused for exploring the next membership level down, thus avoiding redundancy that would appear if cuts were independently explored. New inner points in a cut are obtained as the arithmetic average of adjacent values in intervals forming the cuts of fuzzy input values at the previous level. In this way, each fuzzy input is systematically sampled in a regular way. At each membership level the greatest and least output values form the approximation of the corresponding cut of the fuzzy output; this can be done recursively, using the endpoints of the output cut of the previous higher level and the new evaluated sample points. However, all intermediate results are retained from one level to the next, to avoid recomputing them and for possible subsequent use like sensitivity analyses. This is the backbone of the general transformation method. Note that while, due to its sampling scheme, the transformation method avoids overestimating the output imprecision, it is not immune to underestimating it for functions having a capricious behavior, since only part of the Cartesian product of the supports of fuzzy inputs is scanned by the method.

In Chapter 5, some refinements of the general scheme are suggested. First, when the monotonicity properties of the function under study are partially known, say, the function is monotonic with respect to some variables and nonmonotonic with respect to others, the restricted method can be hybridized with the general one. The latter is only applied to the nonmonotonic variables, for the sake of better efficiency. This is the so-called extended transformation method. Next, it is pointed out that the results of computing the output of the function on the generated sample of points enable a sensitivity analysis of the function to be carried out. More precisely, gain factors for each input variable can be computed from the results and express the relative sensitivity of the output to the variations of each input. This is also useful in providing information about local monotony of the function. More refinements in the computational schemes are proposed, exploiting the structure of the arrays of input sample values, when fuzzy inputs have symmetric membership functions and via the use of sparse grid interpolation techniques.

Interestingly, the author points out, via an example, that the general transformation method can also be applied to solve differential equations with fuzzy coefficients. What the method computes is the fuzzy range of solution functions for all parameter values restricted by the fuzzy intervals. It can be done by fuzzifying an analytical solution, but also, more importantly, by using any numerical solution technique such as a Runge–Kutta method.

Chapter 6 presents additions to fuzzy arithmetic as a miscellany of issues. First it discusses the role of fuzzy arithmetic for the nonprobabilistic handling of uncertainty, and points out the connection with possibility theory [8]. It also stresses the fact that the transformation method is less computationally demanding than a full-fledged Monte Carlo sampling method: fewer points are evaluated, in a nonrandom way. Besides, the author argues that the type of uncertainty encountered in mechanical engineering is not really the one captured by single probability distributions and is more akin to standard interval analysis. However, the book does not account for the existing bridges between possibility theory and probability theory, in terms of imprecise probabilities (see [3] for a bibliography), nor for techniques that combine fuzzy arithmetic and probabilistic methods, on functions having both random and fuzzy arguments [12], [17].

One section is devoted to a preliminary discussion of the potentially important inverse problem with fuzzy intervals: given a set of fuzzy outputs to a set of functions, find the fuzzy range of arguments yielding such fuzzy output intervals. However, the proposal in the book remains ad hoc and unconnected to the existing theoretical works on this topic like Biacino and Lettieri [1] or Friedman, Ming, and Kandel [10]. The impact of the transformation method for computing a defuzzified value representing a fuzzy interval and the degrees of imprecision and of asymmetry (called eccentricity) of a fuzzy interval are also discussed.

The second part of the book proposes several applications of fuzzy interval analysis and the transformation method to mechanical, geotechnical, biomedical, and control engineering problems. I shall not comment on this part, leaving it to specialized engineers to assess its relevance and contribution to the specific case studies. Nevertheless, the presence of detailed accounts of such applications is a very important feature of this book, likely to attract the interest of mechanical or civil engineers and to bring these methods to their research areas.

Final Comments. From the point of view of its contribution to fuzzy set theory, the stress put on the limitation of standard fuzzy arithmetic is a very good asset of the book, likely to draw the attention of fuzzy set researchers to the importance of bridging the gaps between fuzzy interval computation and interval analysis methods (on this topic see [15]). One shortcoming of the book is the chosen style of presentation, which is perhaps both too application-dependent and tends to overuse discretized encodings of fuzzy intervals, close to implementation notations. For instance, the mechanical engineering example in section 3.2, despite its usefulness as an application example, is perhaps not of interest to the general reader and is not really needed to show the limitations of standard fuzzy arithmetic. Simpler, more universal examples can do the job just as well. Section 2 of Chapter 4 provides the algorithm of the transformation method in full generality right away, using complicated notation, based on arrays whose entries are symbols with sophisticated upper and lower indices (e.g., pp. 101, 117), which will hardly be palatable for some readers in a first pass. The transformation method could have been introduced in abstracto, in a more concise and elegant way, at a more basic, application-free, implementation-free level. Despite the concern for practical implementation of fuzzy interval analysis, there is also a relative lack of elementary examples that readers could recompute step by step. The description of sensitivity coefficients and their computation schemes is also hard to grasp at a glance, and needs careful, if not tedious, scrutiny. The scope of the book will make it attractive for mechanical engineers, but may possibly deter others. Despite these debatable presentation choices, this book is a noticeable step toward a mature processing of fuzzy intervals, namely, a better understanding of how to correctly handle them and what to do with them in practice.

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Optimal Stopping and Free-Boundary Problems. *By Goran Peskir and Albert Shiryaev.* Birkhäuser, Basel, 2006. $59.95. xxii+500 pp., hardcover. ISBN 978-3-7643-2419-3.

Optimal stopping problems arise in many fields. One of the most basic is the so-called secretary problem in which one wishes to hire the best secretary from a set of candidates whom one interviews sequentially. However, the hiring rules are such that one has to decide whether to reject or hire each candidate immediately after their interview. That is, one cannot interview all candidates and then choose the best, one has to make an irreversible choice and select a successful candidate at some stage in the interview process. Here an optimal stopping rule provides a number N^*, which states one should interview N^* candidates and then choose the best candidate after those N^*.

In continuous time there are important examples of optimal stopping problems from financial engineering, the most basic of which is the optimal exercise time for an American put. An American put is a contract which allows one to sell some asset S, whose price at time t is S_t, for a fixed price \$$K$ at any time between now and a future time T, the expiration time. The question is: When is the best time to exercise this option, that is, to sell for \$$K$? Does one sell S when its price is \$5 below \$$K$? The contract would then provide a profit of \$5 because one can sell for \$$K$ something whose price is \$$(K-5)$. Or does one hold the contract in the hope that the price will drop even more below K? If the dynamics of S are described by the standard geometric Brownian motion model popular in financial engineering, the solution of this problem is reduced to solving a free boundary problem for a parabolic partial differential equation. That is, there is a critical price curve $S^*(t)$. If at time t the price $S(t)$ is less than $S^*(t)$, one should sell, making a profit of $K - S^*(t)$. If $S(t)$ is greater or equal to $S^*(t)$, one should continue to hold the contract.

Free boundary problems have a long history and are related to determining the boundary between ice and water when ice melts. This is the so-called Stefan problem.

The book under review comprises an extensive study of optimal stopping and related free boundary problems in the framework of the modern theory of stochastic processes. While the emphasis is on continuous time, the discrete time case is also discussed. A classic contribution to discrete time is the 1971 book by Chow, Robbins, and Siegmund [2].

Turning to the book under review, after a general description of the problem in Chapter 1, Chapter 2 provides a review of some of the theory of stochastic processes. Stochastic integrals are briefly defined and statements of the Itô differentiation rule and some of its extensions, such as Tanaka's formula for local time, are given. However, the treatment is rather brief and few proofs are provided.

Chapter 3 presents various optimal stopping and free boundary problems. Methods for their solution are given in Chapter 4. These include the relation of optimal stopping to free boundary problems and methods which involve a time change, space change, or measure change. Optimal stopping problems which arise in the theory of stochastic processes are discussed in Chapter 5 and applications in mathematical statistics, such as detecting properties of a diffusion or Poisson process, are presented in Chapter 6. The final two chapters give applications to finance, including the American put option described above.

The book contains an excellent compilation of problems and descriptions of the field. Its style implies that it will be used more as a reference than as a text. This means it will not usually be read sequentially. Unfortunately, the presentation is often not user friendly for this purpose. In Chapter 2 the concept of uniform integrability is repeatedly used. However, "uniform integrability" does not appear in the index. The notation D appears to represent both cadlag processes and also "the Dirichlet class" on page 556. + and − signs occur in, for example, equations (3.3.30) and (3.5.5). It is unclear what they mean and they do not appear in the list of symbols. Section 6 on page 124 is headed "MLS formulation of optimal stopping problems." However, the reference to "MLS" in the index points just to this section. Only two thirds down the page do we learn that M stands for Mayer, L for Lagrange, and S for supremum.

Unfortunately for beginners in the field this book would be difficult to read. It would be of interest to see more connections drawn between the contents of the book and those in Bensoussan and Lions [1]. Also the nice little book by Wong [4] extends optimal stopping concepts to the case where the process can be stopped only on subsets of the time interval. The neglected work of Davis and Karatzas [3] reduces optimal stopping to the deterministic case by introducing a Lagrange multiplier which enforces the nonanticipative nature of the stopping time.

However, for those with some background in optimal stopping problems the book is a valuable addition to the literature and it should be on the shelf of probabilists, statisticians, and financial engineers.

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Surface Evolution Equations. *By Yoshikazu Giga.* Birkhäuser, Basel, 2006. $119.00. xii+264 pp., hardcover. ISBN 978-3-7643-2430-8.

The level set method has gained incredible popularity since Stanley Osher (UCLA) and James Sethian (UC Berkeley) published the seminal paper on the topic (although there were some previous less-known works). Their original paper, and most of their subsequent papers and books on the topic, focused on the computational aspects of level set and related methods, typically only presenting enough theory to justify the computational approach. That is, the theory was driven by the computation as opposed to vice versa, and as such many elegant mathematical results were omitted simply for the sake of exposition. This new book by Yoshikazu Giga takes a step back and derives a theory of level set methods from the ground up in a more formal mathematical framework than what exists in the works of Osher, Sethian, and their many collaborators.

The book is obviously intended as a mathematical text and does not discuss numerical discretizations or practical implementation issues. Instead, it first presents the basic theory behind surfaces that undergo motion in the direction of the local unit normal including the effects of curvature. After discussing things from this geometric standpoint, the text moves to the theory of viscosity solutions, which is of course a requirement for the level set method and any numerical discretization thereof, but is often taken for granted since researchers in this area typically have training in methods for hyperbolic conservation laws. The author devotes the entire next chapter to the comparison principle, and here one gets a notion of how much he enjoys both the subject matter and the relevant mathematics for its own beauty. Again, this book is not intended for users or implementation, but instead to explore the mathematical foundations behind the method—and in that sense I found the book quite enjoyable to read. After setting the stage in the first three chapters, the level set method is discussed in the second-to-last chapter of the book.

Each of the five chapters concludes with a notes and comments section, which is chatty in nature and briefly sketches out a bit of the published literature. This is a good format that gives the reader a place to start looking for further and related results. There is a community of researchers somewhere in between practioners of pure mathematics and those actively doing numerical computations, and this book will serve that community well. It can also be appreciated by the more computational at heart as long as they have some formal training and ongoing interest in pure mathematics.

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Cooperative Stochastic Differential Games. *By David W. K. Yeung and Leon A. Petrosyan*. Springer-Verlag, New York, 2006. $79.95. xii+242 pp., hardcover. ISBN 987-0387-27260-5.

This interesting book aims at the synthesis of two major branches of game theory: the cooperative and differential games. The former were introduced by von Neumann and Morgenstern in their famous 1944 treatise *The Theory of Games and Economic Behaviour*, while the latter were pioneered by Isaacs from the late 1940s until the publication of his seminal 1965 book *Differential Games*. Separately, these theories flourished over subsequent decades, with cooperative games being studied predominantly by economists and differential games by engineers. The latter trend was not surprising as the classical cooperative games addressed the issue of division of benefits (or costs) of cooperative ventures, whereas differential games focused on controls influencing the dynamics of situations of conflict, such as, for instance, those in pursuit-evasion problems. However, it was clear to many researchers that it would be desirable to develop a unified theory that would combine the cooperative and strategic and dynamic elements of these two important branches of game theory. After all, cooperative ventures are usually conducted in a dynamic way, over time. Conversely, in many contexts, players contributing to controls of complex dynamics will ultimately need to share the benefits or costs of such "cooperation." Despite this, it was also recognized that a variety of both conceptual and technical challenges needed to be overcome to achieve a synthesis of these topics. Of course, these challenges generated new ideas and approaches, many due to the authors of the present book and their associates.

Consequently, Professors Petrosyan and Yeung are making a significant and timely contribution by summarizing both the above-mentioned challenges and an emerging new theory of cooperative (stochastic) dynamic games that addresses these challenges. A key issue that needed resolution can be summarized as follows: whatever solution concept of cooperative game theory players may wish to adopt in a dynamic setting ought to be time consistent (or dynamically stable). Roughly speaking, the rationale used for a division of benefits (or costs) of cooperation at the beginning of the game should still apply at any instant during the course of play until the end of the game. A significant conceptual advance that is used repeatedly in the book is based on the recognition that Isaac's "tenet of transition" or, equivalently, Bellman's "optimality principle" of dynamic programming contains the essence of the above time consistency requirement. This insight also provides the basis for developing mathematical tools for the derivation of dynamically stable solutions to cooperative stochastic differential games.

The book provides a state-of-the-art summary of what has been achieved so far in exploiting the above advance for various classes of dynamic cooperative games including those involving stochasticity. In the latter case, the ordinary differential equations describing the dynamics of the process are replaced by stochastic differential equations. Models involving both finite and infinite time horizons are discussed and a number of interesting examples inspired by applications from economics, resource management, environmental pollution control, and technological change are treated in some detail. Importantly, the authors' analyses often result in computable solutions. In particular, in many cases, exact formulae for "equilibrating" transitory compensation are obtained for specific cooperative game solution concepts. In the case of two-person cooperative stochastic differential games the notion of time consistency leads to a more stringent notion of subgame consistency. The authors derive transitory compensation and payoff distribution procedures under uncertainty and illustrate this technique. Challenging extensions to cooperative stochastic differential games with nontransferable payoffs are also discussed at some length.

The presentation is partially self-contained in the sense that the authors provide an introduction to basic techniques involving dynamic programming, optimal control and stochastic control, and general differential games and their solution concepts and methods. The style is rigorous and derivations of important formulae are presented in a detailed way. The text also includes quite a few problems that may assist students and other researchers entering this topic. However, this book is not for a novice or the faint-hearted; a significant amount of mathematical sophistication and stamina is required.

Because of the complex nature of the subject studied, the mathematical notation is also complex. Similarly, because of the novelty of the topic, a substantial amount of new terminology is introduced. Arguably, the latter reflects an orientation toward dynamic rather than classical cooperative game theory. For instance, a solution concept of cooperative game theory such as the Shapley value is, at times, referred to as an "optimality principle" that in the authors' terminology essentially corresponds to an agreed-upon rationale for the division of benefits of cooperation. It can be expected that the terminology of this fascinating new subject will evolve as its popularity increases.

Overall, this is an important new book on an important—still emerging—topic. I am happy to recommend it to all serious game theory students and researchers as it is likely to stimulate a lot of further developments.

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Mathematical Methods in Robust Control of Linear Stochastic Systems. *By Vasile Dragan, Toader Morozan, and Adrian-Mihail Stoica.* Springer-Verlag, New York, 2006. $79.95. xii+312 pp., hardcover. ISBN 978-0-387-30523-8.

This book considers linear time varying stochastic systems, subjected to white noise disturbances and system parameter Markovian jumping, in the context of optimal control (using H_2 and H_∞ formulations), robust stabilization, and disturbance attenuation. The book has two main objectives, both successfully met: (a) to develop mathematical theory for the above classes of linear time varying systems, and (b) to develop design techniques, including numerical algorithms for solving corresponding problems (in general, given in terms of coupled matrix algebraic Riccati equations). The material presented in the book is organized in seven chapters.

Chapter 1 presents a review of relevant results from probability theory and stochastic differential equations.

Chapter 2 deals with exponential stability in the mean square sense of linear time varying stochastic systems subjected to additive white noise and Markovian parameter jumping. An equivalence of this concept and the exponential stability of a corresponding class of deterministic systems (Lyapunov differential equations) is established.

Concepts of stochastic stabilizability, detectability, observability, and controllability for the considered class of linear stochastic systems are presented in Chapter 3.

In Chapter 4, Riccati equations of stochastic control are studied, including their minimal, maximal, and stabilizing solutions, as well as iterative numerical algorithms for finding such solutions. Both algebraic and differential coupled Riccati equations are considered.

The linear-quadratic optimal control for this class of systems is presented in Chapter 5. H_2-optimal controllers and the tracking problem are studied in detail. The state and control multiplicative white noise problem formulations are also included in this chapter. In addition, the formulations when the performance criterion weighting matrices are not positive (semi)definite are considered.

Robustness properties of stable linear stochastic systems are studied in Chapter 6 via stochastic versions of the bounded real lemma, the small gain theorem, and stability radius theory (robust stability with respect to linear and nonlinear structural uncertainties).

The central part of Chapter 7 is the disturbance attenuation problem. Necessary and sufficient conditions are derived for the existence of stabilizing γ-attenuating controllers (using H_∞ problem formulation) in terms of solutions of game-type coupled Riccati equations and inequalities.

The book is very well written and organized. It is mostly based on the authors' recent research work, which has been already published in very respected journals. In addition, some new research results appear for the first time in this book.

The book is a valuable reference for all researchers and graduate students in applied mathematics and control engineering interested in linear stochastic time varying control systems with Markovian parameter jumping and white noise disturbances.

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Automata Theory with Modern Applications. *By James A. Anderson.* Cambridge University Press, Cambridge, UK, 2006. $45.00. viii+255 pp., softcover. ISBN 978-0-521-61324-8.

Anderson's book gives an excellent, clearly written account of automata theory. The material in Chapters 1–5 deals in a fairly standard way with finite state automata, pushdown automata, and Turing machines, and their connections with regular, context-free, and decidable languages. His approach, however, is outstanding for the clarity of the exposition, the excellent diagrams, the well-chosen worked examples, and the many exercises.

Authors (and indeed publishers) of textbooks have a difficult choice to make regarding solutions of exercises: if they are all solved, the teacher making use of the book has to set new exercises; if none are solved, the self-studying student can be discouraged. Some authors compromise by providing solutions to a selection of the exercises, but it is then often hard to guess the rationale behind the selection process. Interestingly, Anderson offers teachers a set of solutions on a website—an excellent idea, though one suspects that enterprising students will quickly jump the "teachers only" barrier.

Chapter 6, titled "A Visual Approach to Formal Languages," contains less familiar material. It begins with a brief and relatively well-known account of combinatorics on words, such as one might find in [5], but moves on to more recent material due to Tom Head [1]. (The book is subtitled, *With Contributions by Tom Head.*) Given a finite alphabet Σ, a *language* in its most general form is simply a subset of Σ^+, the set of all nonnull words in the alphabet Σ. The details are too complicated to be reproduced here, but the visual approach of the chapter title associates a language L with a sketch consisting of a set of 1 by 1 black and white squares in the upper half plane. For example, if $\Sigma = \{a, b\}$ and $L = \{w \in \Sigma^+ : a \text{ and } b \text{ occur equally often in } w\}$, then the sketch of L consists of a completely black left quadrant and a completely white right quadrant. This chapter is undoubtedly the hardest to read.

The final chapter, "From Biopolymers to Formal Language Theory," is the most intriguing of all. From the epoch-making discovery of the DNA double helix, it did not take mathematicians long to realize that (if one ignores the geometry) a single strand of DNA is a word in the alphabet

$$\begin{cases} A = \text{adenine}, & C = \text{cytosine}, \\ G = \text{guanine}, & T = \text{thymine}. \end{cases}$$

The Crick–Watson pairing, in which A and T pair only with each other and C and G pair only with each other, means that if a single strand is known, its companion strand is also known; thus a strand

$$TTTTGGAAACCTTT$$

has a companion strand

$$AAAACCTTAGGAAA\,.$$

The resulting double strand is denoted by $ttttggaaaccttt$ (or of course by $aaaaccttaggaaa$). Motivated by molecular biology, Anderson exemplifies a "splicing" procedure involving the strand $u = ttttggaaccttt$ and another strand $v = tttggaaccttttt$. Specifically, u and v are "cut," so that $u = u_1u_2$ and $v = v_1v_2$, where $u_1 = ttttgga$ and $v_1 = tttgga$. Two new strands,

$$x = u_1v_2 = ttttggaaccttttt$$

and

$$y = v_1u_2 = tttggaaacttt,$$

are formed. The book ends with a study of splicing rules, schemes, systems, and languages. Among the references are [2], [3], and [4].

In a way Anderson has written two books. Chapters 1–5 constitute a beautifully written textbook on automata, languages, and machines, while Chapters 6 and 7 could be expanded into a valuable research-level monograph. It seems highly likely that advances in molecular biology will benefit from a sound mathematical underlay.

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Models of Mechanics. *By Anders Klarbring*. Springer-Verlag, New York, 2006. $77.00. xii+206 pp., hardcover. ISBN 978-1-4020-4834-0.

Mathematical modeling is usually described as an art rather than a science. *Models of Mechanics*, by Anders Klarbring, shows that there is indeed a science of mathematical modeling. While the book is obviously focused on developing models specifically for mechanics, it has a broader value as a general exposition of mathematical modeling. For this reason, this small book belongs on the bookshelves of mathematical modelers who do not work in mechanics, as well as those that do.

Part I is titled "General Background," but an equally valid title would be "A General Theory of Mathematical Modeling." This theory is constructed around two basic principles. First, the author discusses the relationship between the real world of observation and experiment and the ideal or conceptual world of theory. This should be the starting point of any presentation of mathematical modeling, but it is generally lacking in other texts and nowhere presented as well as it is here. The second key principle is that models are constructed from assumptions having two different levels of validity: universal laws that apply in all settings, and particular laws that supply details that vary according to the setting. The author classifies the variety of particular laws as kinematic constraints, force laws, and constitutive laws, with a full discussion postponed until Chapter 8.

The main portion of the book is divided into two roughly equal parts: a part on geometry and universal laws and a part on particular laws and complete models. Klarbring divides model construction into two stages. In the first stage, one writes down the correct "open scheme," which consists of the three conservation laws (mass, linear momentum, and angular momentum) in the appropriate geometric setting. In practice, one need only choose the right open scheme with which to begin a model. To that end, the five chapters of Part II present the conservation laws in every possible geometry, including systems of discrete particles, one-dimensional continua, and three-dimensional continua, with fluids and solids treated separately.

The second stage of modeling by Klarbring's method is to complete the model by adding particular laws to the appropriate open scheme. The variety of final models arises from the variety of physical settings. Part III begins with a chapter that discusses particular laws in general; like the chapters of Part I, this chapter is of value to modelers who work outside mechanics. The last four chapters present the particular laws needed to complete small displacement models, pipe flow models, and models in three-dimensional fluid and solid mechanics. These are given as examples that are useful in themselves and also serve to illustrate the process of completing models.

The author describes his book as being at an intermediate level, and this warning should be taken seriously. This is not a book for readers who have little prior experience with mathematical modeling in mechanics. Its unified approach gives it levels of generality and abstraction that would be incomprehensible to the beginner, while at the same time providing an attractive synthesis and broadening for readers already familiar with models of particle mechanics, pipe flow, fluid mechanics, and linear elasticity. The author's explanation of the distinction between spatial and material time derivatives is clear, but his notation for the material derivative is somewhat confusing. This is one unfortunate drawback of Klarbring's work, but it is a relatively minor point in comparison with the value of the theoretical framework and generally clear writing.

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Probability: Modeling and Applications to Random Processes. *By Gregory K. Miller.* Wiley-Interscience, Hoboken, NJ, 2006. \$110. xxiv+459 pp., hardcover. ISBN 978-0-471-45892-0.

This is an introductory probability text that is somewhat different from most introductory probability texts. On the one hand, it covers the standard introductory topics (sample spaces, laws of probability, conditioning, discrete and continuous random variables, expected value, basic distributions, joint distributions) for an audience of students knowing multivariable calculus and a bit of matrix algebra. It is suitable for third-year undergraduates majoring in mathematical sciences, computer science, or engineering. On the other hand, it introduces various topics in modeling and stochastic processes as soon as the required probability concepts are introduced. This makes for an engaging text that clearly shows students why probability is important and ubiquitous.

For example, discrete-time Markov chains are first used to illustrate the calculation of probability by conditioning. The definition of a Markov chain is delayed until the next chapter, in which random variables are introduced. The Poisson process is introduced and explored immediately after the Poisson distribution, and revisited once the exponential distribution has been discussed. In between these two sections on the Poisson process, the student learns a bit about the M/M/1 queue and its equilibrium distribution. Brownian motion is discussed on the heels of the normal distribution, and is revisited in a later chapter. The author also devotes significant space to modeling issues: given a sample of data points from an unknown distribution, what distribution fits the model well? Simple parameter estimation is discussed for the main distributions, and the χ^2 goodness-of-fit test is described. Also, the reader is given tastes of a few more specialized topics, including Fisher's exact test, the secretary problem from optimal stopping theory, time series models, and self-avoiding random walks.

This makes for a very full course, but a satisfying one for students who will take only one undergraduate course in probability (e.g., engineering or computer science majors). The author has based it on the course of 15–16 weeks that he teaches at his university. However, my university has courses of 12–13 weeks, so I would not be able to use this book without omitting a lot of material.

Each chapter contains a large number of exercises of various kinds. Some are theoretical, some treat applications, and a few involve data analysis. Most should be within reach of the average student. Some exercises are refreshingly original (e.g., the geometric distribution is illustrated by a high school basketball coach who ends each practice by choosing one boy to shoot free throws and making the team run sprints until the boy succeeds), but not too many of the exercises will really challenge the outstanding students. Occasionally, the applications are less than satisfying, such as the following example, in which the application is irrelevant to the question: "Brownian motion has been used in optimal switching problems which arise in manufacturing when one phase of a production process must be switched on and off due to economic factors. Suppose in a context such as this a Brownian motion process $\{X(t)\}$ with $\sigma^2 = 10$ is implemented. What is the density function associated with $X(t)$? What is the probability that $X(24) < 25$?" The text often says that a particular proof or example will be dealt with in the exercises, but does not give the number of the exercise. Since the longer chapters have 50 to 100 exercises, this is not convenient for the reader. Also, the author chose to number examples and sections but not theorems; thus, for example, an exercise refers to "the last theorem of this chapter," which appears 16 pages earlier. Better cross-referencing would have been worth the trouble.

The author makes an effort to write with a conversational style. This shows up in some places more than others. It usually does not draw attention to itself, and the net effect is positive. An exception is the preface "To the Student," which would probably be fine if spoken by the author to his class on the first day of the term, but on the printed page it tends to come out as condescending.

Regretfully, the book contains a large number of errors. Some are editorial errors, such as misspellings of some mathematicians' names (e.g., James Stirling of the famous formula is written "Sterling"), wrong words ("State and prove and extension..." instead of "an extension"), and missing or superfluous words ("... show that that if ..."). Such errors may not hinder the reader's understanding, but they are irritating and distracting. I am surprised that they were not caught before publication.

There are also more serious errors. References to equations occasionally contain the wrong equation number. There are some improper uses of mathematical terms (e.g., we read "the limit of this sequence approaches one" instead of "the limit equals one" or "the sequence approaches one"). The author uses the fact that the expected value of a sum is the sum of the expected values, but never states this fact explicitly (e.g., as a theorem). An exercise in Chapter 6 requires the fact that $E(XY) = E(X)E(Y)$ for independent random variables X and Y, but this fact is not mentioned in the text until Chapter 8. The definition for "countably infinite" is wrong, since it does not exclude finite sets. An exercise asks for an argument that any state in a Markov chain with $P_{ii} > 0$ is recurrent (which is of course false). The author states that a particular 4-state Markov chain satisfies the conditions for limiting probabilities to exist, but does not notice that the chain is periodic.

In summary, I feel that this would be a useful book for some audiences, such as an intensive third-year probability course for math or engineering students. The modeling and introductory stochastic processes are effectively integrated into the text. Unfortunately, the editing leaves much to be desired. In particular, there are many errors, more than should appear in a textbook. I hope that there will be a second edition or a reprinted version with corrections.

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Understanding and Implementing the Finite Element Method. *By Mark S. Gockenbach.* SIAM, Philadelphia, 2006. $87.00. xvi+363 pp., softcover. ISBN 978-0-898716-14-6.

This book will make an excellent text for an undergraduate applied mathematics class on the numerical solution of partial differential equations by the finite element method. It contains a balanced blend of numerical methods, theory, and implementation considerations. I was particularly pleased to see the treatment of modern techniques, like preconditioned conjugate gradients, multigrid, and adaptive grid refinement, as mainstream topics. This is not a text for a class on programming for scientific computing; it is intended for a class on the finite element method. The accompanying MATLAB programs (available by download from the web) allow students to experiment with aspects of the finite element method without spending hours programming in a language like Fortran or C. A more theoretically oriented numerical analysis class will find the lack of proofs for most theorems disturbing, although references where the proofs can be found are always given. On the other extreme, an engineering class may find the mathematics a little deep, but most of it can be skipped or deemphasized while still learning sufficient basics to understand the practical implementation details.

The book is organized in four parts: the finite element method, implementation, solution of linear systems, and adaptive grid refinement. It focuses on the model 2D linear elliptic boundary value problem

$$\begin{aligned} -\nabla \cdot (\kappa \nabla u) &= f \text{ in } \Omega, \\ u &= g \text{ on } \Gamma_1, \\ \kappa \frac{\partial u}{\partial n} &= h \text{ on } \Gamma_2, \end{aligned}$$

where Ω is a bounded region in the plane with boundary $\Gamma_1 \cup \Gamma_2$, and κ, f, g, and h are scalar functions of x and y. Other problems are mentioned, and sometimes addressed, but the book rightly focuses on the model problem in order to be concrete. Also the finite element method is limited to the Galerkin finite element method with C^0 piecewise polynomials, primarily with triangle meshes.

In Part I, the finite element method is developed. This is a nice presentation beginning with the weak form of the boundary value problem and the relevant Hilbert spaces. The development proceeds through the Galerkin method to the definition of piecewise polynomial spaces, including isoparametric elements. The final chapter in this part analyzes the convergence of the finite element method. The level of theory in this part seems appropriate for an applied mathematics class. The necessary math to provide motivation, develop the methods, and derive the equations is there, but lengthy proofs of theorems are omitted. The exercises at the end of each chapter in this part are mathematical, usually beginning with the word "derive," "show," or "prove."

Part II addresses the practical implementation of the finite element method. The algorithms and mesh data structure are presented in such a way that they could be implemented in a straightforward manner in any procedural language like Fortran or C. The focus is on clarity, not efficiency. The actual implementation of the algorithms is given in MATLAB, and they are available for download. The implementation is limited to the 2D model problem and triangular elements, although the last chapter in this part considers some other problems. Solution of the resulting linear system is postponed until the next part (the MATLAB solver is used at this point), but all other details of the implementation are addressed, including numerical quadrature, high order elements, and isoparametric elements for curved boundaries. The exercises in this part are a mixture of mathematical problems, operation/memory counts, using the given MATLAB programs, and writing new MATLAB programs.

Part III covers the solution of the linear system of equations that is generated by the finite element method. Direct methods are presented to contrast them to iterative methods. Stationary iterative methods are presented so that they can be used as preconditioners and smoothers, and also to contrast the rate of convergence. The preconditioned conjugate gradient and multigrid methods are presented as the preferred solution methods. A limited number of preconditioners are discussed. The hierarchical basis preconditioner is featured, and brief mention is given to diagonal scaling, SSOR, incomplete Cholesky, and fast Poisson solver preconditioners. A straightforward geometric multigrid method is derived using interpolation for the prolongation operator and its transpose for restriction. Much more could have been said about different types of multigrid methods (including algebraic multigrid) and other preconditioners for conjugate gradients, but at least the student gets a taste of modern solution methods for linear systems. Examples are given to illustrate the rate of convergence of these methods. Exercises in this part include both mathematical problems and the use of given MATLAB routines.

Part IV addresses adaptive methods. The focus is on the development of an adaptive mesh refinement algorithm using newest node bisection of triangles. This includes element-by-element error estimators, a strategy for choosing which triangles to refine, and the local refinement of the mesh. Three error estimators are derived. Examples are given to illustrate the advantage of adaptive grids. Exercises in this part are a mixture of mathematical problems, using the given MATLAB programs, and writing new MATLAB programs.

There are many topics not covered in this book, such as parallel implementation, domain decomposition, and 3D problems, and some of the topics covered could have more breadth, such as the presentation of more preconditioners and other multigrid methods. But to do so would distract from the main purpose of the book, which is to present the foundations of the finite element method and how to implement it in a succinct and concrete manner. Overall, I think this book will make an excellent text for its intended audience.

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Computing with hp-Adaptive Finite Elements. Volume I: One and Two Dimensional Elliptic and Maxwell Problems. *By Leszek Demkowicz.* Chapman and Hall/CRC, Boca Raton, FL, 2007. $89.95. xxvi+398 pp., hardcover. ISBN 978-1-58488-671-6.

The finite element method has become part of the standard computational toolbox for solving partial differential equations. However, even with the advent of faster processors, larger memory, and parallel computing, standard off-the-shelf methods may not work. Effective use of computational resources is still essential in solving difficult problems in two and three dimensions. Equally important, reliable results cannot be obtained without intelligent use of these same resources. Adaptive finite element methods offer one possible solution to these two problems while requiring minimal user intervention. There are few texts on adaptive finite element methods. This volume makes a fundamental contribution to the field of *hp*-adaptive finite element methods. It is not a survey or summary of *hp*-adaptive methods but a presentation of the author's approach. Nevertheless, I consider it the best current book in the area. It provides a greater level of detail, especially regarding the algorithms, than one typically finds in finite element texts (and in some spots I would have liked even more detail). The author also develops the theoretical underpinnings that enhance the reliability of his approach. I highly recommend this book for anyone interested in learning about adaptive finite element methods.

Adaptive finite element codes have been under development for almost three decades. Adaptive methods seek to enrich (refine) the discretization where certain features of the solution need to be enhanced and to coarsen it in regions where little of interest is occurring. Two adaptive strategies have become commonplace. They are often referred to as *h*- and *p*-refinement. In *h*-refinement elements are either added or removed. With *p*-refinement the order of approximation on an element is increased or decreased. *p*-refinement offers more rapid convergence but requires higher solution regularity. *h*-refinement is more appropriate in regions where the solution is less smooth. In two and three dimensions anisotropic *h*- and *p*-refinements that align with features of interest are advantageous. *hp*-adaptive methods seek to combine the strengths of both approaches. Work by Babuška and coworkers [5, 6, 7, 8, 9] demonstrated that for a certain class of problems exponential rates of convergence could be achieved with geometrically graded meshes. In practice finding these meshes involves significant effort on the part of the user. The book under review describes the author's effort to achieve these exponential convergence rates automatically.

This can only be done if accurate and efficient methods are available for determining regions needing refinement and/or coarsening. The most common approach is to compute a posteriori error estimates [1, 10]. These estimates often involve calculations over element patches after the solution has been computed. Ideally these estimates should approach, from above, the true error as the grid is refined. Once these estimates are available a procedure must be implemented to determine the next discretization.

This volume makes three important contributions to the development of effective *hp*-adaptive methods: (i) a flexible data structure that supports anisotropic *hp*-refinements in one and two dimensions (with a natural extension to three dimensions); (ii) an effective strategy for determining error estimates for a large number of refinement choices; and (iii) a clever method for determining the next discretization. These contributions are valuable even for readers whose interest is in problems not directly discussed in the text.

The tone of the book is familiar, at times almost folksy. There are the usual grammatical errors and oversights in a first edition. I found some sections a bit terse. It is divided into three roughly equal parts. The first seven chapters describe the one-dimensional algorithm. The next nine chapters extend this approach to linear elliptic equations in two dimensions. The remaining five chapters focus on applying the method to Maxwell equations in two dimensions. Accompanying the book is a CD containing one- and two-dimensional codes. There are frequent references to these codes in the text. It is the author's belief that to gain a full understanding of the material the reader must actively engage with the computer program (written primarily in Fortran 90). The author uses the first twelve chapters as a text in his graduate-level introductory finite element course.

The first three chapters cover material that can be found in any modern text on the finite element method. They include a description of the model problem, weak formulation, Sobolev spaces, and the Ritz–Galerkin method. The finite element grid and basis are also introduced. High order is presented right away based on Lobatto polynomials on the master element. Practical aspects such as the computation and assembly of the element matrices are described.

The fourth chapter introduces the reader to the one-dimensional code with the goal of writing a processor routine to interface with it. A sample processor routine *mysolver.F* is provided in the directory *mysolver*.

The next two chapters cover the three key components of the author's approach. The typical adaptive finite element code relies on complicated data structures that allow easy access to element connectivity during assembly. The author has chosen a simple data structure that utilizes sophisticated routines to reconstruct the connectivity. This simplicity means that complicated anisotropic grids in both h and p can be stored and modified.

The *hp*-adaptive strategy is based on a sequence of coarse/fine grid pairs. The fine grid is generated from the coarse grid by dividing each element in half and increasing the order by one over the coarse grid element. Solutions are computed on both grids. For each element in the coarse grid estimates of the error are generated on a set of h- and p-refinements of the element. These estimates are computed using a technique developed by the author called projection-based interpolation. The next coarse grid is created by selecting the best refinement for each element. The procedure for choosing the set of refinements is also described. Although this two-grid procedure is more expensive than more traditional a posteriori error estimation strategies, it leads to more optimal grids.

Chapter 7 applies the algorithms of the first six chapters to wave propagation problems.

The second portion of the book is devoted to extending the one-dimensional *hp*-techniques to elliptic problems in two dimensions. Attention is paid to aspects of the method that are more complicated in two dimensions. Model problems are discussed in Chapter 8. An entire chapter (Chapter 9) is devoted to Sobolev spaces including embedding and trace theorems. Basis functions for both triangular and quadrilateral elements are detailed in Chapter 10. Chapter 11 introduces the two-dimensional code. The author's geometric modeling strategy and the mesh generation algorithm are presented in Chapter 12. The h-refinement strategy in two dimensions is complicated by the presence of hanging or constrained nodes and edges. The data structure for storing the grid remains the same as in one dimension but the algorithms for accessing connectivity information are now more complex. These issues are dealt with in Chapter 13. The automatic method for selecting h and p in Chapter 14 is a natural extension from one dimension. Applications of the two-dimensional algorithm appear in the next two chapters including exterior boundary-value problems.

The main contribution of the last part of the book is the introduction of De Rham diagrams. These commutative diagrams have become effective tools in the development of stable bases for mixed finite element methods [2]. The Maxwell equations provide an excellent setting to introduce these tools. The equations are given in Chapter 17. The De Rham diagram and consistent finite element bases are developed in Chapter 18. Chapters 19 and 20 extend the hp-algorithm to these equations. The last chapter presents examples.

Although I was able to easily download the codes to several computers I have access to, I was unable to compile them using the author's makefiles despite assistance from several systems administrators. We had a variety of difficulties with compilers (the bulk of the code is written in Fortran 90) and library locations that we were unable to resolve. This was the most disappointing aspect of the book and one I hope the author will address.

Computational and theoretical exercises accompany almost all chapters so the book could be used as a text for one or more courses. Despite the author's experience I would not recommend the book for an introductory finite element course, primarily because it is too narrowly focused on his approach. It would be more suitable for a more advanced finite element course as it introduces many important current research topics. The exercises will also benefit readers who wish to understand hp-adaptive finite element methods.

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Riemannian Geometry: A Modern Introduction. Second Edition. *By Isaac Chavel.* Cambridge University Press, Cambridge, UK, 2006. $45.00. xvi+471 pp., softcover. ISBN 978-0-521-61954-7.

Perelman's stunning recent proof of the 100-year-old Poincaré conjecture in topology principally used methods of Riemannian geometry and analysis, specifically the so-called Ricci flow. Chavel's excellent text on Riemannian geometry is therefore of special current interest.

The Poincaré Conjecture. It has long been known that among connected compact two-dimensional manifolds, such as the sphere, the torus, and the two-holed torus, the sphere is characterized by the fact that any loop can be contracted to a point, whereas a loop around a torus, for example, cannot be contracted to a point. The Poincaré conjecture, suggested by Henri Poincaré in 1904, proposes the analogous result for three-dimensional manifolds. At the 2006 International Congress of Mathematicians, Grigori Perelman was awarded the Fields Medal for its proof, although he declined to accept it. The basic idea of the proof, due to Richard Hamilton, is to start with any such three-manifold and let it shrink at each point in each direction at a rate proportional to its so-called Ricci curvature. If you can show that you eventually end up with a round sphere, you can conclude that you must have started with a (deformed) sphere.

Curvature. Riemannian geometry could be called the study of manifolds and curvature. Even for a two-dimensional surface in Euclidean three space, the curvature is a bit complicated, because the surface can have different curvatures in different directions. The maximum and minimum (upward) curvatures are called the *principal curvatures* and they occur in orthogonal directions. At the bottom of an ellipsoid, both are positive. At the top of an ellipsoid, both are negative. On a saddle, one is positive and one is negative. Their average is called the *mean curvature*. Their product is called the *Gauss curvature*. All of these curvatures are apparently "extrinsic," i.e., depend on how the surface curves in the surrounding space. Gauss's Theorema Egregium ("remarkable theorem") says that the Gauss curvature happens to be "intrinsic," i.e., can be measured by a two-dimensional creature living inside the surface.

For higher-dimensional manifolds, there is really no new kind of intrinsic curvature, just the Gauss curvature of two-dimensional sections, called the *sectional curvature*. Of course at any point you can take the average of all the sectional curvatures to get a single number, called the *scalar curvature* (usually renormalized by a dimensional constant); or, given any one direction at a point, you can take a normalized average over all sections containing that direction, called the *Ricci curvature*.

The Ricci curvature turns out to be the curvature of greatest physical significance. Much of general relativity can be derived from Einstein's assumption that the Ricci curvature of empty space-time is zero. Ricci curvature is the curvature used in Perelman's proof of the Poincaré conjecture. It is the basic geometric quantity that tells you how space spreads out or contracts as you move in some direction; it appears, for example, in the so-called second variation formula for the second derivative of the area of a hypersurface as you move it.

Chavel. Chavel's text provides an excellent modern treatment of Riemannian geometry. As in most modern treatments and unlike the discussion above following my own text, the relevant curvatures are so defined as to be obviously intrinsic. This makes the definitions more abstract but in some sense more natural.

Chavel's first two chapters provide the basic concepts. The third chapter moves on to volume comparison theorems. For example, Bishop's theorem says that if a connected compact manifold has Ricci curvature greater than the sphere's, then its volume is less than the sphere's. The Bonnet–Myers theorem concludes that the diameter is less than the sphere's. Cheng's theorem treats the case of equality. An appendix discusses eigenvalue comparison theorems. The second half of the book takes up advanced topics. Chapter 4 studies the rate of volume growth and relates it to similar properties of the fundamental group, which is built from loops under equivalence under deformation and contraction. For a sphere, since every loop can be contracted to a point, the fundamental group is trivial and the total volume is finite. More interesting are manifolds with infinite fundamental groups and infinite volume. Chavel includes a nice explanation of how to approximate a manifold by finitely many well-distributed points.

Chapter 5, the principal new addition in the 2006 second edition, focuses on two-dimensional manifolds and provides a thorough treatment of the isoperimetric problem, which seeks the least-perimeter way to enclose a given amount of area or volume. It was not until 1996 that Benjamini and Cao proved that geodesic balls provide the least-perimeter way to enclose a given area in the two-dimensional paraboloid $\{z = x^2 + y^2\}$. The proof uses curve-shortening flow, a simpler precursor of Ricci flow. The analogous result for higher-dimensional paraboloids remains an open question today. Chapters 6 and 8 study the isoperimetric problem in higher-dimensional manifolds, including some deeper results not readily accessible elsewhere. The isoperimetric theorem for $\mathbf{R}^\nu$ says that the least area $A(V)$ required to enclose volume V is that of a round ball, proportional to $V^{(\nu-1)/\nu}$. As shown by Federer and Fleming, such isoperimetric inequalities are equivalent to "Sobolev inequalities" relating the L^1 norm of the gradient of a function to the $L^{\nu/(\nu-1)}$ norm of the function. (The proof uses the very useful generalization of Fubini's theorem called the co-area formula.) By studying the constants of proportionality for various values of the "dimension" ν, including the so-called Cheeger's constant for the case $\nu = \infty$, one obtains important estimates, often related to the curvature of the manifold. The celebrated Levy–Gromov theorem says that if a manifold has Ricci curvature greater than a round sphere's, then under suitable normalizations, the least area required to enclose volume V is greater than on the sphere. Chavel also presents the isoperimetric inequalities of Croke and Buser. Chapter 7 treats "the kinematic density."

The final chapter, Chapter 9, on "Comparison and finiteness theorems," starts with curvature-based estimates of Rauch and Heinze-Karcher and concludes with Cheeger's finiteness theorem, which says roughly that given bounds on the sectional curvature, an upper bound on the diameter, and a lower bound on the volume, there are only finitely many n-dimensional manifolds.

Each chapter concludes with an excellent section of notes and advanced exercises with further results, with hints and sketches of solutions at the end of the book. Chavel's book would make for a difficult graduate text, but I think that it is the best reference on Riemannian geometry available, especially for someone interested in isoperimetric problems. It is essentially free of errors. A fraction the size of Spivak's older *Comprehensive Introduction to Differential Geometry* (Publish or Perish, 1999), this book provides an insightful modern perspective on topics of current research interest. Petersen's *Riemannian Geometry* (which I have heard also has a new second edition) has more examples, more computations, and more exercises for students. Chavel's text is one of about a dozen mathematics books I keep at home for ready reference.

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Measure Theory and Probability Theory. *By Krishna B. Athreya and Soumendra N. Lahiri.* Springer-Verlag, New York, 2006. $89.95. xviii+618 pp., hardcover. ISBN 978-0387-32903-1.

Probabilists have a special relationship to measure theory. Whereas mathematicians may often view measure theory mostly through its applications to Lebesgue measure on Euclidean spaces, probabilists routinely also deal with spaces of sequences, trees, functions, and other objects where the relevant measures can be understood only from a solid foundation in general measure theory. This divide is reflected in how measure theory is being taught to graduate students; those focusing on mathematics usually learn measure theory from courses in real analysis and may use, for example, texts by Folland or Royden (both titled *Real Analysis*). On the other hand, students who focus on probability and statistics usually learn their measure theory from courses in advanced probability theory where real analysis plays an important, yet smaller, role, the stress being instead on the construction of probability measures, expectation as an integral, and conditional expectation as a Radon–Nikodým derivative. Some standard texts are Chung, *A Course in Probability Theory*; Billingsley, *Probability and Measure*; and Resnick, *A Probability Path*.

Anybody who has taught a course in measure-based probability has faced the problem of how to start. Should one start with probability measures and develop the general theory later or should one start with general measures and introduce probability as a special case? Billingsley is an example of somebody who favors the first approach and, although in many ways a very good text, it becomes a bit clunky when many of the results and arguments for probability measures are repeated and extended for general measures. In *A Modern Approach to Probability Theory*, Fristedt and Gray take the same route but hold off with proofs of the central theorems until the sections on general measure. Other advocates of the first approach are Chung, who in the third edition of his classic text puts measure theory in an appendix, and Resnick, who skips general measures altogether.

It is of course a matter of taste but I decidedly favor the second approach, to go from the general to the specific, provided that the two prime special cases—probabilities and Lebesgue measure—are brought in early as concrete examples of measures. This philosophy is implemented in a recent addition to the literature, Krishna Athreya and Soumendra Lahiri's *Measure Theory and Probability Theory*. Almost encyclopedic in scope, the book provides not only extensive coverage of standard measure theory, real and functional analysis, and graduate level probability theory, but also Markov chains, renewal theory, and Brownian motion, as well as more recent topics such as the Black–Scholes formula, Markov chain Monte Carlo methods, and bootstrapping. The competence of the authors is unquestionable. Both have impressive publication records, currently adding a combined total of about 150 items to MathSciNet, and both remain active researchers and are also experienced teachers. Their fields of expertise are represented in the book in sections on branching processes (Athreya) and bootstrapping (Lahiri).

The style of writing is clear and precise, combining the Eurasian tradition of stringency and attention to detail with the New World demand for relevance and applicability. The text progresses at a fairly rapid pace but the authors nevertheless strive to give intuitive explanations of the various concepts and also provide an ample supply of examples and problems.

Its wide range of topics and results makes *Measure Theory and Probability Theory* not only a splendid textbook but also a nice addition to any probabilist's reference library. I view the book from the point of view of a probabilist, but it should be pointed out that the first five chapters can be used for a pure mathematics course in measure theory and real analysis without any reference to probability. Whether you are an instructor who has not found a text to your liking or wishes to try something new, a researcher in need of a reference work, or just somebody who wants to learn some measure theory to lighten up your life, *Measure Theory and Probability Theory* is an excellent text that I highly recommend.

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Differential Equations: An Introduction to Modern Methods & Applications. *By James R. Brannan and William E. Boyce.* John Wiley & Sons, Hoboken, NJ, 2007. $127.95. xvii+682 pp., hardcover. ISBN 978-0-471-65141-3.

The publication of this new differential equation book by Brannan and Boyce marks a substantial departure from the original version of Boyce and DiPrima, published by Wiley in 1965. Boyce and DiPrima's *Elementary Differential Equations* and its extended version *Elementary Differential Equations and Boundary Value Problems* have presumably sold millions of copies in the intervening years, with their eighth editions appearing in 2005. Today's grandparents and their grandchildren are likely to have learned their DiffEQs from the same classic text.

In 1965, Bill Boyce and Dick DiPrima were among the lively and relatively young faculty teaching engineering and science students at Rennselaer Polytechnic Institute. Their enduring success as best-selling authors was well earned. The books feature unusually clear writing, significant substance appropriate to their audience, and good problems. The eighth edition looks much different from the first, with wide margins, color graphics, and a CD copy of the software ODE Architect, yet the basic coverage is mainly unchanged, except for a long-present chapter on nonlinear differential equations and stability. Bill has continued to launch new editions of Boyce and DiPrima since Dick's death in 1984, and he's been assisted by others, including Jim Brannan of Clemson University, in writing the associated solution manuals.

Boyce and DiPrima intended to avoid the cookbook feel of many old differential equation books. They emphasized the fundamental role of the existence-uniqueness theorem for initial value problems and even included its proof. They also included a fairly thorough coverage of series solutions near ordinary and regular singular points, along with some treatment of associated Bessel and other special functions. Their numerical methods chapter, despite the gigantic changes in practical computing and available software since 1965, has even held its relevance. In recent editions, numerically obtained results nicely show the convergence of partial sums. Matrices were lightly used in the first edition, but there was no discussion of the matrix exponential and its asymptotic behavior, which is extensively described in later editions. The eighty-page nonlinear chapter now includes phase plane analysis, predator-prey models, limit cycles, and chaos and strange attractors.

Brannan and Boyce emphasize a systems approach to differential equations and include chapter projects graded according to their level of difficulty in terms of both analysis and computation. ODE Architect is made available through the WileyPLUS website. There's no coverage of power series solutions and possibly less orientation toward two-point boundary value problems. The Laplace transform chapter is more relevant than in Boyce and DiPrima, due to later connections to control theory. In summary, the new book is comparable to the eighth edition of *Elementary Differential Equations*, though there are differences in emphasis.

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Partial Differential Equations. Second Edition. *By Jürgen Jost.* Springer-Verlag, New York, 2007. \$59.95. xiv+356 pp., hardcover. ISBN 978-0-387-49318-3.

The original edition of this book was reviewed by the same reviewer in *SIAM Review*, 45 (2003), pp. 376–377. From the original review:

> This book provides an introduction primarily to the theory of scalar second-order elliptic partial differential equations. It is organized around the basic theme of techniques for existence proofs. Four types of such techniques are developed: those based on the maximum principle; those based on solving parabolic equations and letting time tend to infinity; those based on variational methods; and those based on continuation methods.... The author has deliberately decided to focus on scalar second-order elliptic equations and discuss them in depth. In the process, he develops many of the important techniques in partial differential equations. The decision of choosing this book as a text for a course is largely a matter of whether one wants to follow the same choice of topics.

The new edition differs primarily by the addition of a chapter on reaction-diffusion systems. The first two sections of this chapter develop basic existence results and the connection between a priori bounds and global existence and stability. A third section focuses on the Turing instability.

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Nonsmooth Variational Problems and Their Inequalities: Comparison Principles and Applications. *By Siegfried Carl, Vy Khoi Le, and Dumitru Motreanu.* Springer-Verlag, New York, 2007. $89.95. xii+395 pp., hardcover. Springer Monographs in Mathematics. ISBN 978-0-387-30653-7.

Boundary value problems for elliptic partial differential equations have played an important role in mathematically modeling many physical phenomena. Most often these problems arise as Euler–Lagrange equations for certain energy functionals given by the physical process being modeled and thus there is an intimate relationship between variational methods and the existence theory for solutions of these boundary value problems. One very early such problem is that of finding a function, harmonic in a given domain, which satisfies given boundary data on the boundary of the domain. This problem was successfully tackled for special kinds of domains (balls) by Poisson and a very elegant solution was found by Perron with his use of sub- and superharmonic functions [14]. His method, nowadays called the Perron process, not only provides for the existence of solutions in a generalized sense, but also may be considered to be a constructive process, thus making it amenable to obtaining approximate solutions together with error bounds (see, e.g., [7]).

The problem considered by Perron is, of course, a very special one; however, it proved to provide the basis of a method that allows for treatment of highly nonlinear problems via the method of sub- and supersolutions (often also called lower and upper solutions). Early work in this direction goes back to the 1930s work of Scorza Dragoni [18] and Nagumo [12, 13]. In fact, Nagumo's work proved to be the starting point for a whole school of mathematicians whose investigations yielded many important contributions (see, e.g., [1, 11], among others). At about the same time others started to devote themselves to the study of such problems, e.g., Knobloch [10], Jackson and Schrader [9], this reviewer [16], Sattinger [15], and Amann and Crandall [2] (for a review of the state of the art in the 1970s see [17]). The common core of their results is that if an ordered pair of sub- and supersolutions exists, classical solutions are obtained and approximated by the ordered pair.

It was in the middle of the 1970s that Hess [8] and Deuel and Hess [5] started to reformulate these problems in a variational setting and proved comparison principles in a weak sense, thus, in some sense, returning the problems to their proper setting, as Euler–Lagrange variational problems, and making available the immense progress that had been achieved by this time in the areas of linear and nonlinear functional analysis.

It is in this spirit that the present monograph develops in a careful and thorough way the theory of sub- and supersolutions for variational equalities and inequalities.

The book starts with a well-developed introduction giving a broad overview of the subject and then delivers in a concise way all the important mathematical tools needed for the study (Sobolev space theory; the theory of monotone and pseudomonotone operators; Leray–Lions operators; existence results for evolution equations and inequalities; nonsmooth analysis). It then proceeds to give a detailed discussion of the use of sub-supersolution techniques for variational equations, both single- and multivalued. This is followed by a lengthy part (three chapters) on variational and hemivariational inequalities.

As already mentioned, the material is developed very carefully and includes (often from a fresh and new point of view) many by now classical results about various kinds of boundary value problems (such as Dirichlet, Neumann, Robin, no-flux, unilateral, etc.) for nonlinear elliptic and parabolic problems.

All three authors have made significant contributions to the area in the past few years and much of their own work is included, as is the work of many others who have worked on such problems.

The book is an important addition to the literature on nonlinear analysis, convex analysis, variational and hemivariational inequalities, nonlinear elliptic and parabolic partial differential equations, elasticity theory, fracture mechanics, and general obstacle and unilateral problems; it will be a welcome addition to the libraries of researchers and students of these areas.

Other recent books which are concerned with such inequality techniques and applications are [4] and [6], the first being mainly concerned with boundary value problems for nonlinear ordinary differential equations and the second with nonlinear elliptic partial differential equations. A monograph which makes extensive use of sub- and supersolution techniques (in a classical sense) to study nonlinear evolution problems arising in combustion theory is [3].

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Measurement in Fluid Mechanics. *By Stavros Tavoularis*. Cambridge University Press, Cambridge, UK, 2005. $85.00. xiv+354 pp., hardcover. ISBN 978-0-521-81518-5.

As the author points out in his preface, making measurements in fluid mechanics requires a deep appreciation of fluid mechanics and a wide variety of practical skills. Experimental fluid mechanics is a synthetic process, rather like "design," and it is equally difficult to teach well. This new book is intended as a textbook for graduate students, although it is accessible to upper-level undergraduates and could easily be used in a self-study program by other professionals. It provides a first-class introduction to the subject.

The book is based on the author's experience teaching a graduate course on this topic at the University of Ottawa. As would be expected from a textbook (but very unusual to find in this genre), each chapter in the book concludes with a series of questions and problems that could be used for homework assignments or programmed study. Given its relatively short length, the book covers a remarkable amount of material. The introductory chapters on the general properties of measurement systems (introducing their static and dynamic response), measurement uncertainty (a crucial but often neglected aspect of measurement), and signal conditioning and data acquisition are especially welcome, and would be useful to any experimentalist, regardless of their particular field of interest. Another very useful chapter provides extensive background material on experiments using optical techniques, and a shorter, less successful chapter gives some rules for planning experiments. The rest of the book (occupying about 50% of its entire length) is more specifically directed to experiments in fluid mechanics, and covers more-or-less standard topics (measurement of flow pressure, flow rate, flow velocity, temperature, composition, and wall shear stress), with an additional chapter on flow visualization techniques.

In each chapter, the author strikes an excellent balance between breadth and depth. He avoids rather neatly the temptation to write a minireview of each topic and provides a state-of-the-art description of the instrument or the measurement method, together with a list of important references where more in-depth material, or technical and historical background, may be found. These references are thoughtfully chosen and up-to-date, and reflect well on the author's deep knowledge of the field.

Of course, it is always possible to find fault. For instance, I would have liked to have seen a more comprehensive treatment of techniques for compressible flows, reacting flows, and a fuller treatment of spectroscopic techniques. A more extensive reference list on these topics may have satisfactorily covered the gaps, and in this respect it was disappointing to see no reference to Pope & Goin's excellent *High-Speed Wind Tunnel Testing* (Wiley, 1965, reprinted by Robert Krieger, 1978). But these are relatively minor criticisms of a book that will undoubtedly find many fans.

How does it stack up against the competition? Surprisingly, there is not much competition out there. Goldstein's *Fluid Mechanics Measurements* (Taylor & Francis, 2nd ed., 1996) is probably the only serious contender, and because that book is a compilation of chapters by different authors it is by comparison not as consistent or unified. Goldstein also lacks much of the introductory material given here, and has no homework exercises. On the other hand, Goldstein covers more material (including measurements in two-phase flows and non-Newtonian flows), but it comes at a price: Amazon.com lists Goldstein at $215 compared to this book's relatively modest price of $85.

As far as the rest of the field is concerned, Bradshaw's *Experimental Fluid Mechanics* (Pergamon Press, 2nd ed., 1970) and *An Introduction to Turbulence and its Measurement* (Pergamon Press, 1971) are (unfortunately) both long out of print and outdated. Other books, such as Rae & Pope's *Low-Speed Wind Tunnel Testing* (Wiley, 1984) and Perry's *Hot-Wire Anemometry* (OUP, 1982), are more specialized and not suitable as textbooks. The book fills an obvious gap in the literature and serves its intended audience very well. While it is not perfect, it succeeds in succinctly summarizing the enormous breadth of experimental fluid mechanics without losing its depth. I am very happy to add this volume to my reference shelf, and will certainly buy another copy for my students to use in the laboratory, recommending it to them as the first stop on the way to better measurements.

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