

Math Stat, HW1, Due: Friday 1/24/2014

Practice problem:

1. Let X_1, X_2, \dots, X_n be a random sample from a uniform distribution on $[0, \theta]$ and let $\hat{\theta} = 2\bar{X}$.

- (a) Show that $\hat{\theta}$ is unbiased.
- (b) Find the variance of $\hat{\theta}$.

Turn-in problems

1. Let X_1, \dots, X_n be a sample from a uniform distribution on $[\theta, 1]$ where θ is unknown.

- (a) Find an unbiased estimator $\hat{\theta}$ based on the sample mean \bar{X} .
- (b) Compute the variance of $\hat{\theta}$.

2. Let X_1, X_2 be a sample of size $n = 2$ from a distribution with mean μ and variance σ^2 and consider the linear estimator

$$L = aX_1 + (1 - a)X_2$$

where $0 \leq a \leq 1$.

- (a) Show that L is unbiased for any value of a .
- (b) For what value of a in $[0, 1]$ is the variance of L minimized? Use minimization techniques from calculus.