Math Stat, HW1, Due: Friday 1/24/2014

Practice problem:

1. Let $X_1, X_2, ..., X_n$ be a random sample from a uniform distribution on $[0, \theta]$ and let $\hat{\theta} = 2\bar{X}$.

   (a) Show that $\hat{\theta}$ is unbiased.
   (b) Find the variance of $\hat{\theta}$.

Turn-in problems

1. Let $X_1, ..., X_n$ be a sample from a uniform distribution on $[\theta, 1]$ where $\theta$ is unknown.

   (a) Find an unbiased estimator $\hat{\theta}$ based on the sample mean $\bar{X}$.
   (b) Compute the variance of $\hat{\theta}$.

2. Let $X_1, X_2$ be a sample of size $n = 2$ from a distribution with mean $\mu$ and variance $\sigma^2$ and consider the linear estimator

   $$L = aX_1 + (1 - a)X_2$$

   where $0 \leq a \leq 1$.

   (a) Show that $L$ is unbiased for any value of $a$.
   (b) For what value of $a$ in $[0, 1]$ is the variance of $L$ minimized? Use minimization techniques from calculus.