Math Stat, HW2, due January 31

Turn-in problems

1(a) Let $X_1, ..., X_n$ be a sample from a uniform distribution on $[\theta, 1]$ where θ is unknown. Find an unbiased estimator $\widehat{\theta}$ based on the sample mean $\bar{X}$. Also find the value of $\widehat{\theta}$ and its estimated standard error if $n = 5$ and you have the observations $-1.0, 0.4, -0.3, 0.7, -0.9$.

(b) Inspired by the example we did in class for the unif$[0, \theta]$ distribution, suggest an unbiased estimator $\widetilde{\theta}$ that is more efficient that $\widehat{\theta}$ from (a). You may use the intuitive idea that in a sample of size n from a uniform distribution, the "average picture" is that the observations are equidistantly spread over the interval (like we pointed out in class to argue that $E[X_{(n)}] = \theta n/(n+1)$ when the observations are from a unif$[0, \theta]$ distribution).

2. Recall that the sample variance s^2 is an unbiased estimator of the variance σ^2 for any distribution. However, the sample standard deviation s is not unbiased for σ. In fact, it can be shown that s underestimates σ; thus, show that $E[s] < \sigma$ (hint available upon request!).