Math Stat, HW2, due January 31

Turn-in problems

1(a) Let $X_1, ..., X_n$ be a sample from a uniform distribution on $[\theta, 1]$ where $\theta$ is unknown. Find an unbiased estimator $\hat{\theta}$ based on the sample mean $\bar{X}$. Also find the value of $\hat{\theta}$ and its estimated standard error if $n = 5$ and you have the observations $-1.0, 0.4, -0.3, 0.7, -0.9$.

(b) Inspired by the example we did in class for the unif[0, $\theta$] distribution, suggest an unbiased estimator $\tilde{\theta}$ that is more efficient than $\hat{\theta}$ from (a). You may use the intuitive idea that in a sample of size $n$ from a uniform distribution, the “average picture” is that the observations are equidistantly spread over the interval (like we pointed out in class to argue that $E[X_{(n)}] = \theta n/(n + 1)$ when the observations are from a unif[0, $\theta$] distribution).

2. Recall that the sample variance $s^2$ is an unbiased estimator of the variance $\sigma^2$ for any distribution. However, the sample standard deviation $s$ is not unbiased for $\sigma$. In fact, it can be shown that $s$ underestimates $\sigma$; thus, show that $E[s] < \sigma$ (hint available upon request!).