Math Stat, HW4, due Monday 2/24

Practice problems:

Book, p.380: 18

Book, p.436: 1, 9

1(a) To investigate whether caffeine effects cholesterol levels, five patients had their cholesterol levels measured before and after taking doses of caffeine. Suppose that individual cholesterol levels follow a normal distribution and find a symmetric 95% confidence interval for the difference in cholesterol level based on the following data:

Before:	162,	168,	197,	202,	225
After:	179,	170,	196,	188,	210

Note: You need to consider the difference "after minus before" for each patient. Why are these values normally distributed?

(b) If somebody were to claim that caffeine effects cholesterol levels, do the current data support the claim?

2. Let $Z_1, Z_2, ...$ be independent random variables that are $N(0,1)$. Does the random variable T below have a t distribution?

$$T = \frac{2Z_1}{\sqrt{Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2}}$$

Turn-in problems

1. Let $X_1, ..., X_n$ be a random sample from a uniform distribution on $[0, \theta]$ where θ is unknown and let $\widehat{\theta} = \dfrac{n+1}{n} X_{(n)}$.

(a) Find a *one-sided* confidence interval of the type $\theta \leq T_3$ (as opposed to a two-sided interval of the type $T_1 \leq \theta \leq T_2$ that we did in class) with confidence level q. (The notation T_3 is to emphasize that it will be different from

T_2. Also note that this one-sided interval is *upper bounded*; we could also have a lower bounded one-wided interval of the type $\theta \geq T_4$.)

(b) Why can't you simply remove the lower bound in the interval $T_1 \leq \theta \leq T_2 \quad (q)$ to get $\theta \leq T_2 \quad (q)$ (same T_2)?

2. Let $Z_1, Z_2, ...$ be independent random variables that are $N(0,1)$. For each of the following random variables, determine whether it has a t distribution and if it does, find the degrees of freedom.

(a) $\dfrac{Z_1}{\sqrt{\dfrac{Z_1^2 + Z_2^2 + Z_3^2}{3}}}$

(b) $\dfrac{Z_1}{|Z_2|}$

(c) $\dfrac{Z_1 + Z_2}{\sqrt{Z_3^2 + Z_4^2}}$

3(a) Below are seven measurements of the ozone level (in ppm) taken at an environmental measuring station. Suppose that these have a normal distribution and find a 95% symmetric confidence interval for the mean μ.

0.06, 0.07, 0.08, 0.11, 0.12, 0.14, 0.21

(b) If the mean ozone level exceeds 0.10 ppm it is considered unhealthy. In order not to scare the public unnecessarily, a warning is only sent out when there is "95% certainty" that the critical level is exceeded. Do the current data warrant a warning?