Math Stat, HW6, nothing to turn in

1. Birds leave a feeding station and it is studied in which direction they leave. Each bird is classified as going N(orth), S(outh), W(est), or E(ast). It has been suggested that birds choose the direction randomly. Based on the data below, formulate and test the appropriate hypotheses.

Direction	N	S	W	E
Number of birds	199	169	177	220

2. Let $X_1, ..., X_n$ be $N(\mu, \sigma^2)$ where σ^2 is known. If we test

$$H_0 : \mu = \mu_0 \;\; vs. \;\; H_A : \mu > \mu_0$$

we base the test on

$$T = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

which is $N(0, 1)$ under H_0, and reject on level α if $T > c$ where $\Phi(c) = 1 - \alpha$. To find the power function $g(\mu)$, we need the distribution of T for an arbitrary value μ.

(a) Show that $T \sim N(\frac{\mu - \mu_0}{\sigma/\sqrt{n}}, 1)$.
(b) Use the distribution in **(a)** to show that the power function $g(\mu)$ satisfies

$$g(\mu) = 1 - \Phi\left(c - \frac{\mu - \mu_0}{\sigma/\sqrt{n}}\right)$$

where Φ is, as usual, the standard normal cdf.

(c) Suppose that $\sigma^2 = 1$ and find the probability to reject $H_0 : \mu = 10$ on level 5% if the sample size is $n = 6$, and the true value of μ is 11. In other words, find $g(11)$.

3. In a study of voting behavior in the 2000 presidential election, it was investigated how voting was associated with education level. Use the contingency table below to test on the 5% level whether voting is independent of education level.

	No degree	High school	Some college	College degree
Voter	26	223	215	387
Nonvoter	168	432	221	211

4. Let p be the unknown probability of an event that may occur in a trial. You repeat the trial n times and get n successes and no failures. Assume that the prior distribution of p is uniform on $[0, 1]$.

(a) Find the pdf of the posterior distribution (by direct computation, not by using results about the beta distribution).
(b) Find the Bayes estimator. Compare to the relative frequency.

5. Microchips are being produced, and there is a certain probability p that a chip is defective and thus useless. To estimate p, 100 chips are checked and 4 of these are defective. In the archives you find the results of four previous studies, where the estimated values of p are $0.05, 0.06, 0.08$, and 0.10.

(a) Suggest a way to use this information to choose a prior beta distribution.
(b) From part **(a)** it follows that a reasonable prior is $B(9.8, 125.9)$. Find the posterior distribution and the Bayes estimate. Compare with the relative frequency.