Probability, Solutions to HW1

Practice problems:

10. Only (b) is consistent with the axioms: (a) and (d) violate Proposition 1.3(c), and (c) violates Proposition 1.3(d) (it has \( P(A \cap B) > P(A) \) although \( A \cap B \subseteq A \)).

11. If the total weight exceeds 20, at least one fish must weigh more than 10. Thus, if \( C \) occurs, either \( A \) or \( B \) must occur, that is, \( A \cup B \) must occur. Hence, \( C \subseteq A \cup B \).

15. The Venn diagram does not depict the most general case. For example, there is no intersection between \( A \) and \( D \) that does not intersect either \( B \) or \( C \). Thus, the Venn diagram depicts a special case in which the intersections \( A \cap D \cap C^c \cap B^c \) and \( B \cap C \cap A^c \cap D^c \) are empty. No general formulas can be inferred from this Venn diagram. Note that the most general Venn diagram of \( n \) events has \( 2^n \) disjoint pieces (counting the one outside the events). Think about why this is the case!