Test 1, Probability. Due in class Friday 10/7

1. True or false? Give proofs (Venn diagrams are fine where possible) or counterexamples.

(a) $P(A \cap B^c) = P(A) - P(A \cap B)$

(b) $P(A \cap B) = P(A)P(B)$

(c) $P(A \cup B) \geq P(A) + P(B) - 1$

(d) $P(A|B^c) = 1 - P(A|B)$

(e) $P(A|B) \leq P(A)$

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2. Let A and B be events, with $P(A) = 1/3$ and $P(B) = 1/4$. Compute both $P(A \cup B)$ and $P(A \cap B)$ if

(a) A and B are independent
(b) A and B are disjoint
(c) A^c and B are disjoint.

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3. You are dealt a bridge hand (13 cards from a regular deck of 52 cards). Find a combinatorial expression for the probability that your hand contains 5 hearts, 3 diamonds, and no aces (you do not need to compute it).

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4. In order to have a certain genetic disorder, you need to have one copy of the gene A and one copy of the gene B, where having either gene is independent of having the other. Gene A and gene B each occurs at a frequency of 1% in the population. There is a test for gene A and another test for gene B.

Each test has the property that if you have the gene, the test result is always positive, and if you do not have it, there is a 1% chance that the test will still be positive in error. A group of people are tested for gene A; those that test negative are declared healthy and those that test positive are tested for gene B.

(a) What percentage of people have gene A but not the disorder?

(b) What percentage of those who test positive for gene A actually have gene A?

(c) Among those who go on to take the second test and test positive there as well, what percentage has the disorder?

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5. Four letters are chosen at random from the word ALABAMA, then put back randomly into the four empty spaces. What is the probability of getting the correct text back?

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6. A computer executes jobs one by one and if it is busy, incoming jobs wait in a queue with room for at most two jobs. If the queue is full, incoming jobs are lost, and if a job is under execution at any time, the probability is 2/3 that this job is finished before a new job arrives. The period when there are jobs in the system is called a "busy period." Suppose that there are currently two jobs in the system (one job being executed and another waiting in the queue). What is the probability that no incoming jobs are lost before the current busy period ends?