**Test 2, Probability, due in class 11/21**

**1.** True or false? Give proofs, counterexamples, or intuitive arguments.

**(a)** If $f$ is a pmf, then $f(x) \leq 1$ for all $x$.

**(b)** If $f$ is a pdf, then $f(x) \leq 1$ for all $x$.

**(c)** If $F$ is the cdf of a discrete random variable, then $F(x) \leq 1$ for all $x$.

**(d)** If $F$ is the cdf of a continuous random variable, then $F(x) \leq 1$ for all $x$.

**(e)** If $X \sim \text{bin}(n, p)$, then $n - X \sim \text{bin}(n, 1 - p)$.

**(f)** If $X \sim \text{bin}(n, p)$, then $2X \sim \text{bin}(2n, p)$.

**(g)** If $(X, Y)$ is jointly continuous with range $[0, 1] \times [0, 1]$, then $X$ and $Y$ are independent.

**(h)** If $(X, Y)$ is jointly continuous on a triangle, then $X$ and $Y$ can not be independent.

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**2.** Compute $P(X = 0)$ and $P(0 < X \leq 2)$ if $X$ has

**(a)** a geometric distribution with variance 6

**(b)** an exponential distribution with variance $1/4$

**(c)** a Poisson distribution with variance 2

**(d)** a normal distribution with mean 1 and variance 2

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**3.** Consider a sphere with volume $V$ and radius $R$ where $R$ has pdf

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$$f(x) = cx^2, \quad 0 \le x \le 2$$

**(a)** Find the constant $c$.
**(b)** Compute the mean and variance of $V$.

In case you forgot: $V = 4\pi R^3/3$.

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**4.** During hurricane season (6 months from June through November), there is about a 50-50 chance that the Gulf Coast will be hit by at least one hurricane. Use time unit "months" (for simplicity you can assume that all months are of equal length) and assume a Poisson process with rate $\lambda$.

**(a)** Find the value of $\lambda$
**(b)** Let $X$ be the number of hurricanes that hit the Gulf Coast during the months of August and September. What is the distribution of $X$ (name and parameter(s))?
**(c)** If there have been no hits for the last 2 seasons, what is the probability that the first hit of the current season comes in June?

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**5.** Let $(X, Y)$ be uniform on the region where $0 \le x \le 1$ and $x^2 \le y \le x$. Find

**(a)** the joint pdf of $(X, Y)$ (and sketch the range of $(X, Y)$)
**(b)** $P(X > 1/2)$
**(c)** The marginal pdf's $f_X(x)$ and $f_Y(y)$ (including the ranges of $X$ and $Y$).
**(d)** The conditional pdf's $f_X(x|y)$ and $f_Y(y|x)$ (including ranges).
**(e)** Are any of the distributions in **(d)** uniform?

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**6(a)** Let $X$ be a continuous random variable with cdf $F$ and pdf $f$. Show that $E[F(X)] = 1/2$.

**(b)** Let $(X, Y)$ be jointly continuous and such that $X$ and $Y$ are independent and have the same marginal distributions (thus, the same cdf $F$ and pdf $f$). Use the result in **(a)** to show that $P(Y \leq X) = 1/2$.

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**7.** Let $(X, Y)$ be jointly continuous with range $R^2$, joint cdf $F$, and joint pdf $f$.

**(a)** Sketch the region whose probability equals $F(2, 2) - F(1, 1) + F(0, 0)$.

**(b)** Let $m = \min(X, Y)$ and $M = \max(X, Y)$ and let $g$ be the joint pdf of $(m, M)$. Express $g$ in terms of $f$.