Probability, HW3

Practice problems:

1. Book, p.72–: 87, 90, 93

2. In turn-in problem 1 on HW3, if you buy a car and it develops problems, should you go back and punch the dealer in the nose? Compute the relevant conditional probability.

Turn-in problems:

1. You have estimated that there is a 5% chance that something is wrong with your car each time you start it. If something is wrong, there is a 75% chance that the "check engine" light comes on. If nothing is wrong, there is still a 10% chance that the light comes on in error. If the light comes on, what is the probability that there is something wrong?

2. A certain illness occurs with a frequency of 2% in a population. There is a test for the illness that is 100% accurate for those that have the illness and is such that 95% of those who test positive have the illness. If you are healthy, what is the probability that you will test positive?

3(a) Ann, Bob, and Carol each tells the truth with probability 1/3 and lies otherwise, independently of each other. If Ann says that Bob claims that Carol told the truth, what is the probability Carol told the truth?

Note: Some tacit assumptions must be made: Carol makes a statement that is either true or false and Bob knows which it is. He then tells Ann either "Carol told the truth" or "Carol lied."

(b) Add a 4th person and show that the probability is now 13/41.

(c) Now suppose we have n people. What do you think happens to the probability as $n \to \infty$? (It is not e^{-1} this time...)

Extra credit problem 1: In the last problem, suppose we have $n+1$ people where n is even. Show that the probability is

$$\frac{\sum_{j=0}^{n/2} \binom{n}{2j} 2^{2j}}{\sum_{j=0}^{n/2} \binom{n}{2j} 2^{2j} + 2 \sum_{j=0}^{n/2-1} \binom{n}{2j+1} 2^{2j+1}}$$

(with a similar expression for odd n).

Extra credit problem 2: Let $A_1, ..., A_n$ be any events. Prove *Bonferroni's inequality*:

$$P\left(\bigcup_{k=1}^{n} A_k\right) \leq \sum_{k=1}^{n} P(A_k)$$