1. True or false? No proofs, arguments, or examples needed.

(a) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$

(b) If $\rho(X, Y) = 0$, then X and Y are independent.

(c) $f_X(x) \leq f(x, y)$

(d) $F(x, y) \leq F_X(x)$

(e) If X and Y are independent, $\text{Var}[XY] = \text{Var}[X]\text{Var}[Y]$

(f) If $X \geq 0$ is a continuous random variable and A is the area of a square with side X, then $E[A] > E[X]^2$.

(g) If $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$, then X and Y are uncorrelated.

(h) If X is any random variable, then $\rho(X, X^2) = 1$.

(i) If X is any random variable, then $\rho(X, -X) = -1$.

(j) If $\text{Var}[X] \leq 1$ and $\text{Var}[Y] \leq 1$, then $\text{Cov}[X, Y] \leq 1$

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2. Compute $P(X = 1)$, $P(X > 1)$, and $P(X^2 > 1)$ if X has

(a) a binomial distribution with mean 4 and variance 2.

(b) an exponential distribution with variance 1.

(c) a Poisson distribution with variance 1

(d) a normal distribution with mean 0 and variance 2

(e) the pdf $f(x) = 0.75x(2 - x),\ 0 \leq x \leq 2$

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3. A voice recognition computer program can correctly determine the gender of a given speaker with 80% probability. In a group of people where there are twice as many males as females, a person is selected at random and is determined by the program to be male. What is the probability that the speaker was male?

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4. Let A be the area of a disk with radius R where R has pdf $f(x) = 4x^3$, $0 \leq x \leq 1$. Find the pdf and the mean of A.

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5. Visits to a website occur according to a Poisson process such that the expected time between visits is 30 seconds.

(a) What is the probability of at least 2 visits in a given minute?
(b) If the last visit was 2 minutes ago, what is the probability that the next visit comes within a minute?
(c) In a ten-minute period, each minute is classified as "bad" if there are no visits and "good" otherwise. Let X be the number of good minutes. What is the distribution of X (name and parameter(s)).

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6. (X, Y) has a uniform distribution on the triangle with corners at $(0, 0)$, $(0, 1)$, and $(1, 0)$.

(a) Find $P(X > 2Y)$
(b) Find the marginal pdfs f_X and f_Y.
(c) Find the correlation coefficient between X and $X + Y$.

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7. A lake contains two species of fish, A and B, and there are twice as many A-fish as B-fish. The A-fish have weights that have mean 100 and standard

deviation 20, and the B-fish have weights that have mean 10 and standard deviation 2. Let W denote the weight of a randomly selected fish and compute the mean and variance of W.

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8. Draw a cartoon that captures the essence of the class. Artistic quality is appreciated but not required.