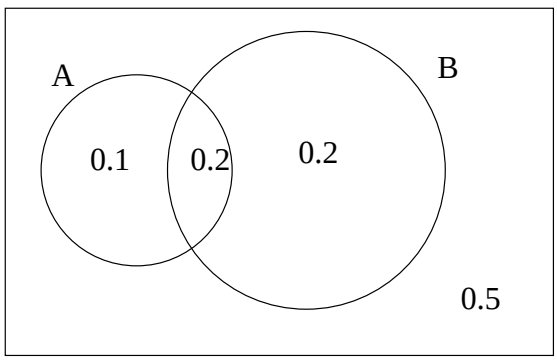
Probabilistic Models, Solutions 1

Practice problems

1. Both **(c)** and **(e)** can be used. Note that **(e)** is more detailed as it takes into account the entire sequence of heads and tails. For example, the outcome "2" in **(c)** corresponds to the event $\{HHT, HTH, THH\}$ in **(e)**.

2. Sample space $\Omega = \{H, T, E\}$ where $P(E) = 0.1$ and $P(H) = 2P(T)$. As $P(H) + P(T) + P(E) = 1$, we get the equation $3P(T) = 0.9$ which gives $P(T) = 0.3$ and $P(H) = 0.6$.

3.



(a) $P(B) = 0.4$
(b,c) $P(A \cap B^c) = 0.1$
(d) $P(A^c) = 0.7$
(e) $P(B^c) = 0.6$
(f) $P(A^c \cap B^c) = 0.5$ (which also equals $P((A \cup B)')$)
(g) $P(A \cup B^c) = 0.1 + 0.2 + 0.5 = 0.8$ (anything that is in A or outside B
(h) $P((A \cap B) \cup (A^c \cap B^c) = 0.2 + 0.5 = 0.7$ (anything that is in both A and B or in neither of them)

4(a) $26^3 \cdot 10^2 = 1,757,600$
(b) $26 \cdot 25 \cdot 24 \cdot 10^2 = 1,560,000$
(c) $26^3 \cdot 10 \cdot 9 = 1,581,840$
(d) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 = 1,404,000$

(e) Number of ways to choose 3 letters in order: $26 \cdot 25 \cdot 24$

Number of ways to choose 2 digits in order: $10 \cdot 9$

Number of ways to choose positions for the 2 digits: $\binom{5}{2} = 10$

Total number of ways: $\binom{5}{2} \cdot 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 = 14,040,000$

5. If A occurs, B must also occur, hence $P(B|A) = 1$. If B occurs, 4 out of 5 equally likely outcomes are even, hence $P(A|B) = 3/5$.

6(a) If the first roll gives 5, the second must give 3 to get sum 8, hence $P(E|F) = 1/6$. **(b)** If the sum equals 8, we have one of the 5 equally likely outcomes $(2,6), (3,5), (4,4), (5,3), (6,2)$ and hence $P(F|E) = 1/5$.

7(a) $P(A \cap B) = P(A)P(B) = 1/8$, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/4 + 1/2 - 1/8 = 5/8$
(b) $P(A \cap B) = 0$ (since $A \cap B = \emptyset$), $P(A \cup B) = P(A) + P(B) = 3/4$

Turn-in problems

1(a) 1/365 **(b)** $\binom{50}{2} = \dfrac{50 \cdot 49}{2} = 1225.$

2. The number of ways to get the distribution 5–4–3–1 for a fixed suit order is $\binom{13}{5}\binom{13}{4}\binom{13}{3}\binom{13}{1}$. There are 4! different suit orders (or, if you wish, $\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1}$) and we get

$$P(5-4-3-1) = \frac{4!\binom{13}{5}\binom{13}{4}\binom{13}{3}\binom{13}{1}}{\binom{52}{13}} \approx 0.13$$

and hence 5–4–3–1 is more likely than 4–3–3–3.

3. Let A and B be the events that he wins the first and second elections, respectively. Then

(a) $P(A \cap B) = P(A)P(B|A) = 0.60 \cdot 0.75 = 0.45$
(b) $P(A \cap B^c) = P(A)P(B^c|A) = 0.60 \cdot 0.25 = 0.15$

4(a) $P(A \cap B \cap C) = 0$ but $P(A)P(B)P(C) = 1/8$. Note that the events

are pairwise independent.

(b) For example, A and C are not independent since $P(A) = 1/2$ but $P(A|C) = 1$. Note that $P(A \cap B \cap C) = P(A)P(B)P(C) = 1/36$.