Probability Models, Test 1, due February 23

1(a) In what sense is Orr's model a probability model? In what additional sense is our model a probability model?
(b) In our model, we choose K out of N nodes at random. Another way of thinking of it is to choose the nodes one by one. In the first step, each node has probability $1/N$ to be chosen; in the second step, each node has probability $1/(N-1)$ to be chosen, and so on, in each step choosing a node according to a uniform distribution. We could generalize this to choosing nodes according to distributions other than the uniform. What would this mean from a biological point of view?

2. Consider the yeast network with $K = 20$ and $p = 0.01$.

(a) Compute the speciation probability in our model.
(b) By how many percent does the speciation probability increase if we we double K?
(c) By how many percent must we increase K in order to double the speciation probability?
(d) Compute the speciation probability in Orr's model.
(e) With $p = 0.01$, how many substitutions would we need in our model to get a speciation probability that is as high as Orr's in **(d)**?
(f) What does the Finnish word *lumipallo* mean?

3. Suppose we study a particular bacterial population where we know that each gene interaction has a 50/50 chance of leading to an incompatibility. We also know that there is on average 1 substitution every 20 generations. If 30 populations are studied for 100 generations each and 16 of them experience no speciation events, what is the estimated density of the network?

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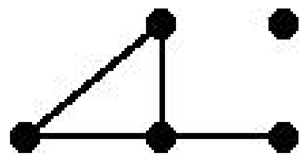
4. For the network below, let X be the number of interactions (edges) when we choose $K = 3$ nodes. Find **(a)** $P(X = j)$ for $j = 0, 1, 2, 3$, **(b)** $E[X]$ and $\mathrm{Var}[X]$, **(c)** the density α, **(d)** the speciation probability, both exactly and with our approximation formula.



5. Recall the quantities N_S : the number of edge pairs that share a node and N_D : the number of edge pairs that do not share a node. Find N_S and N_D in the following networks:

(a) The complete network with N nodes. *Hint:* Argue that $N_S = 3\binom{N}{3}$ and establish a similar type of expression for N_D.

(b) The disjoint network with N nodes.

(c) This network: