Probabilistic Models, Homework 2, due January 31

Practice problems:

1. You have estimated that there is a 5% chance that something is wrong with your car each time you start it. If something is wrong, there is a 75% chance that the "check engine" light comes on. If nothing is wrong, there is still a 10% chance that the light comes on in error. What is the probability that the light comes on when you start your car?

2. Draw cards repeatedly from a deck of cards and let X be the number of draws until you get the ace of spades. Find the range and pmf of X if you draw **(a)** with replacement **(b)** without replacement.

3. In the game chuck-a-luck you wager \$1 and roll 3 dice. If no 6s show, you lose. If k 6s show for $k = 1, 2, 3$, you win $\$k$. Find your expected gain.

4. Suppose that the probability of a rainy day in Seattle on an arbitrary day in December is 0.8. If a day is not rainy, call it sunny. In which of the following cases is it reasonable to assume a binomial distribution? Argue why/why not and give the parameter values n and p where you have a binomial distribution.

(a) You count the number of rainy days on Christmas Eve for ten consecutive years.
(b) You count the number of rainy days in December next year.
(c) You count the number of rainy days on the first of each month for a year.
(d) You count the number of sunny days on Christmas Eve for ten consecutive years.

Turn-in problems:

1. If you wager \$1 on a straight bet (payout 35:1) or on an odd bet (payout 1:1) in Roulette, the expected gain $-2/38$, as seen in class.

(a) In a *split bet*, you bet on two numbers and if any of these two come up,

you win. What should the payout be in order to give the same expected gain as with a straight bet?
(b) Almost all roulette bets have the same expected gain of $-2/38$. There is however one exception: the "five number bet" in which you win if any of the numbers 00, 0, 1, 2, or 3 comes up and the payout is 6:1. Let X be your gain if you wager \$1 and find $E[X]$.
(c) What would the payout in a five number bet have to be in order to get the usual expected gain of $-2/38$?

2. Ann and Bob flip a fair coin 4 times. Each time it shows heads, Ann gets a point; otherwise Bob gets a point.

(a) What is the distribution of Ann's final score (name and parameters)?
(b) What is the most likely final result?
(c) Which is more likely: that it ends 2–2 or that somebody wins 3–1?

3. Let X be a discrete random variable whose range is a subset of the nonnegative integers $\{0, 1, 2, ...\}$ (the range itself can be finite or infinite). It can be shown that

$$E[X] = \sum_{k=0}^{\infty} P(X > k)$$

which provides an alternative way of computing expected values for that is sometimes easier than using the defitition.

(a) Verify that the formula is correct if X is the result of a die roll.

(b) Use indicators to prove the formula (remember, if I is the indicator of an event A, then $E[I] = P(A)$).