Probabilistic Models, Homework 3, due February 14

Turn-in problems:

1. Consider the yeast network which has $N = 6,018$ and $\alpha = 0.0087$, and let $K = 10$. Compute the speciation probability in our model and in Orr’s model, and compare the two if $p = 0.1$. Comment on qualitative and quantitative aspects of “snowballing.”

2. For each of the three networks below, let $X$ be the number of interactions (edges) when we choose $K = 3$ nodes. Note that all networks have $N = 6$ and $N_E = 5$.

(a) For each network, find $P(X = j), j = 0, 1, 2, 3$ (easier to use brute force than trying sophisticated combinatorics).

(b) For each network, compute $E[X]$ both using (a) and the expression $\alpha \binom{K}{2}$ (and make sure they coincide).

(c) For each network, compute the speciation probability $E[S]$ for $p = 0.1$, both exactly using (a) and with our approximation formula.

(d) For each network, compute the variance $\text{Var}[X]$ using (a).

(e) In (c), find the difference (without sign) between the exact and approximate answers and plot them against the variances in (d). Comment!
3. For the network below, find $N, N_E, N_S,$ and $N_D$. 