

PROBABILISTIC MODELS, SPRING 2012, FINAL EXAM

1(a) Consider the yeast network with $p = 0.01$. Compute the speciation probability in our model for $K = 10$ and $K = 20$.

(b) Draw a cartoon of a snowman who says something illustrative in regards to **(a)**.

2. The formula

$$3 \left(\binom{N}{3} + \binom{N}{4} \right) = \binom{\binom{N}{2}}{2}$$

is true for any positive integer N . Based on what we know about networks, give an argument for why the formula is true, without any calculations. For partial credit, prove it mathematically.

3. Consider the Wright–Fisher model with $2N = 6$ with 4 copies of A and 2 copies of a in generation 0.

(a) Find the probability that A becomes fixed in the first generation.

(b) Find the probability that a becomes fixed in the first generation.

(c) If the population in **(a)** doubles between the 0th and first generation instead of staying constant, what are the average population size and the effective population size?

4. Find the extinction probability for the following branching processes.

(a) An individual has 1 offspring with probability p and 0 offspring with probability $1 - p$ where $p < 1$.

(b) An individual has 0 offspring with probability $1/4$, 1 offspring with probability $1/4$, or 2 offspring with probability $1/2$.

(c) The offspring distribution has pgf $G(s) = e^{s-1}$.

(d) An individual has 0 offspring with probability $999/1000$ and 1000 offspring with probability $1/1000$.

(e) The offspring distribution has pgf $G(s) = 0.2s + 0.8s^4$.

5. Consider a branching process started from one ancestor and denote its extinction probability by q . Now instead consider the same branching process started from a number Z_0 of ancestors who initiate independent branching processes.

(a) If Z_0 is a fixed integer, what is the extinction probability of the population?

(b) If Z_0 is a random variable with pgf $H(s)$, argue that the extinction probability of the population is $H(q)$.

6. Write a one-page poem about the course that includes the words “speciation,” “heterozygosity,” and “extinction” as well as rhymes with each of these words. Rhymes can be incomplete or approximate.