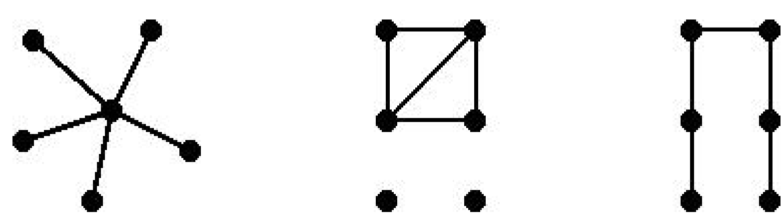
Probability Models, HW2, due February 7

1. Consider the yeast network which has $N = 6,018$ and $\alpha = 0.0087$, let $K = 10$ and assume $p = 0.1$.
(a) Compute the (approximate) speciation probability in our model.
(b) Compute the speciation probability in Orr's model.
(c) Roughly how many substitutions do we need in order to get a speciation probability as high as Orr's? You can use an approximation for $\binom{K}{2}$.

2. For each of the three networks below, let X be the number of interactions (edges) when we choose $K = 3$ nodes. Note that all networks have $N = 6$ and $N_E = 5$.
(a) For each network, find $P(X = j), j = 0, 1, 2, 3$ (easier to use brute force than trying sophisticated combinatorics).
(b) For each network, compute $E[X]$ both using the definition and the probabilities in **(a)** and the expression $\alpha\binom{K}{2}$ (and make sure they coincide).
(c) For each network, compute the speciation probability for $p = 0.1$, both exactly using **(a)** and with our approximation formula.



3. Consider the disjoint network with N nodes, $N_E = N/2$ edges, choose K nodes at random, and let X be the number of edges in our subgraph. Consider the probability $P(X = 0)$.
(a) What is $P(X = 0)$ if $K > N_E$?
(b) What is $P(X = 0)$ if $N = 6$ and $K = 2$?
(c) Find a general expression for $P(X = 0)$ if $K \leq N_E$. Remember that there are $\binom{N}{K}$ ways to choose K nodes out of N so you need to figure out how many of these avoid getting any edges.