Probabilisty Models, Homework 4, due April 7

Turn-in problems:

1(a) Consider the Wright–Fisher model with $2N = 4$ with 3 A's and 1 a in the initial generation 0 (that is, $X_0 = 3$). Find the probability that A becomes fixed in the first generation and in the second generation, respectively (note: if it *becomes* fixed in the first generation, it cannot *become* fixed in the second).

(b) Find the probability that A is fixed in the second generation (note the difference between "is" and "becomes" fixed). Use LTP and condition on the number X_1 of As in the first generation.

(c) Find the probability that fixation of one allele occurs in the first generation.

(d) Now assume variable population size with initial size $2N = 4$ as in **(a)**. Do part **(c)** in the cases where population size in the first generation halves and doubles, respectively (sampling is still done with replacement from the initial population).

2. In the Wright–Fisher model individuals reproduce by sampling $2N$ times with replacement, thus creating the next generation from the present. Another way of looking at this is to note that each individual has a number of offspring that is described by a random variable X which can take on any value between 0 and $2N$. If we label the individuals $1, 2, ..., 2N$, we then have the numbers of offspring $X_1, X_2, ...X_{2N}$ where $X_1 + X_2 + \cdots + X_{2N} = 2N$.

(a) Argue that each X_k has a binomial distribution and give the two parameters.

(b) Argue that the random variables $X_1, X_2, ..., X_{2N}$ are not independent.

(c) Now instead assume that we start with $2N$ individuals and let each individual reproduce according to X_k as above but where the X_k are independent. The size of the next generation is then a random variable. What is its range and distribution?