Probability Models, practice problems

1. Let X be a random variable which takes on the values $0, 1$, and 2 with equal probabilities (uniform distribution on $\{0, 1, 2\}$).

(a) Find the pgf of X.
(b) Use the pgf in **(a)** to find the mean of X.
(c) Let Y be independent of X and have the same distribution as X. Consider $X + Y$ which has range $\{0, 1, 2, 3, 4\}$. Find the pgf of $X + Y$.
(d) Use the pgf in **(c)** to find $P(X + Y = 0)$.

2. Consider a cell population where cells die with probability $1/3$ and divide with probability $2/3$.

(a) Find the extinction probability of a population started from a single ancestor.
(b) Find the extinction probability of a population started from two ancestors.
(c) If you can choose the number of ancestors and want the extinction probability to be at most 1%, how many ancestors should you have?

3. Find the extinction probabilities for the following branching processes.

(a) Cells divide with probability 0.2 and die with probability 0.8.

(b) An individual has 1 offspring with probability p and 0 offspring with probability $1 - p$ where $p < 1$.

(c) The offspring distribution has pgf $G(s) = e^{s-1}$.

(d) The offspring distribution has pgf $G(s) = 0.2s + 0.8s^4$.