PROBABILITY MODELS, FINAL EXAM, due May 9

1. Consider the yeast network with $p = 0.01$. Compute the (approximate) speciation probability in our model and in Orr's model for $K = 10$ and $K = 20$.

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2. Consider the Wright–Fisher model with $2N = 8$ with 2 copies of A and 6 copies of a in generation 0.

(a) Find the probability that A becomes fixed in the first generation.

(b) Find the probability that a becomes fixed in the first generation.

(c) What is the probability that A eventually becomes fixed?

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3. Recall the formula

$$E[T] \approx \frac{4N}{\sigma^2} \quad \text{(generations)}$$

from Homework 5, where σ^2 is the limit of the variance in the offspring distribution as $N \to \infty$.

(a) The formula is valid only if the limit of $\mathrm{Var}[X_k]$ exists. For a case where the formula does not apply, suppose one individual is chosen randomly and given $2N$ offspring; all other individuals are given 0 offspring. Show that the limit of $\mathrm{Var}[X_k]$ does not exist (it equals ∞).

(b) In the situation described in **(a)**, what can you say about the actual time T?

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4. Find the extinction probability for the following branching processes.

(a) The offspring distribution has pgf $G(s) = e^{-0.6(1-s)}, 0 \leq s \leq 1$.

(b) An individual has 0 offspring with probability 1/4, 1 offspring with probability 1/4, or 2 offspring with probability 1/2.

(c) The offspring distribution has pgf $G(s) = \frac{1}{1000}(s + s^2 + \cdots s^{1000})$.

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5. The three main topics we covered in the course were (1) Speciation, (2) the Wright–Fisher Model, and (3) branching processes. Write a poem about the course that includes all these topics. Any style is acceptable (limerick, haiku, iambic pentameter, free form, sonnet,...).