1. Which of the following statements are always true?

(a) $P(A \cap B^c) = P(A) - P(A \cap B)$
(b) $P(A \cap B) = P(A)P(B)$
(c) $P(A) \leq P(A \cup B)$
(d) $P(A|B^c) = 1 - P(A|B)$
(e) If $P(A \cap B \cap C) = P(A)P(B)P(C)$, then A, B, and C are independent.
(f) If A, B, and C are independent, then $P(A \cap B \cap C) = P(A)P(B)P(C)$.

2. Let A and B be events, with $P(A) = 1/4$ and $P(B) = 1/2$. Compute both $P(A \cup B)$ and $P(A \cap B)$ if

(a) A and B are independent
(b) A and B are disjoint
(c) A and B^c are disjoint.

3. You are dealt a bridge hand (13 cards from a regular deck of 52 cards). Find a combinatorial expression for the probability that your hand contains 5 hearts, 3 diamonds, and no aces.

4. You test for a very rare disease that only one in ten thousand people have. If you have the disease the test will always be positive. If you do not have the disease the test is 99% accurate.

(a) What is the probability that you test positive?
(b) If you test positive, what is the probability that you have the disease?

5. A computer executes jobs one by one and if it is busy, incoming jobs wait in a queue with room for at most two jobs. If the queue is full, incoming jobs are lost, and if a job is under execution at any time, the probability is 2/3 that this job is finished before a new job arrives. The period when there are jobs in the system is called a "busy period." Suppose that there are currently two jobs in the system (one job being executed and another waiting in the queue). What is the probability that no incoming jobs are lost before the current busy period ends?

6. The random variable X has cdf

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/4 & \text{for } 1 \le x < 2 \\ 3/4 & \text{for } 2 \le x < 3 \\ 1 & \text{for } x \ge 3 \end{cases}$$

Find **(a)** $P(X = 1)$ **(b)** $P(X = 2)$ **(c)** $P(X = 2.5)$ **(d)** $P(X \le 2.5)$