Stochastic Processes, HW2


Turn-in problems, due Wednesday 2/4

1. Consider the queueing example we did in class where arrivals and departures are equally as likely but suppose that there is only room for 3 customers to wait in line (thus, the maximum number of customers in the system is 4 and if the system is full, arriving customers are turned away). Give the transition graph, the transition matrix, and find the stationary distribution.

2(a) Find the stationary distribution in Problem 4 on HW1.

(b) If you have done (a) correctly, $\pi_0$ is equal to $\pi_1$. Explain why this makes intuitive sense.

3. Consider a Markov chain on some state space $S$ that has two different stationary distributions $\pi$ and $\tau$. Let $\alpha$ be a real number in $[0, 1]$ and define the vector $\nu$ as the convex linear combination $\nu = \alpha \pi + (1 - \alpha)\tau$. Show that $\nu$ is a stationary distribution for any such $\alpha$ (remember that there are two properties you need to establish). Thus, a Markov chain has either 0, 1, or (uncountably) infinitely many stationary distributions.