Stochastic Processes, HW4

1. Consider a random walk that can not only step up or down but also stay where it is. Suppose that is steps up with probability \( p > 0 \), down with probability \( q > 0 \), and stays where it is with probability \( r \) where \( p + q + r = 1 \).

Formally, \( S_n = \sum_{k=1}^{n} X_k \) where the \( X_k \) are i.i.d. assuming the values 1, 0 and -1 with probabilities \( p, r, \) and \( q \), respectively.

Describe this random walk in a transition graph. For what values of \( p, q, r \) is it irreducible? Aperiodic? Recurrent?


Turn-in problems, due Wednesday 3/25

1(a) In problem 1 above, suppose that \( p < q \). Find \( P_0(\tau_1 < \infty) \) expressed in terms of \( p \) and \( q \) (condition on the first step).

(b) In the same problem, suppose that \( p > q \). Find \( E_0[\tau_1] \) expressed in terms of \( p \) and \( q \) (condition on the first step and remember that \( r = 1 - p - q \)).


3(a) Let \( X \) have a geometric distribution on \( \{1, 2, \ldots\} \) and let \( Y \) have a geometric distribution on \( \{0, 1, 2, \ldots\} \). Find the pgf’s \( G_X \) and \( G_Y \).

(b) The answers in (a) should satisfy the relation \( G_X(s) = sG_Y(s) \). Explain this relation by using the result for the pgf of a sum of independent random variables (remember that constants are also random variables with probability 1 for one value).