Stochastic Processes, HW6

Turn-in problems, due Monday 4/13

1. A continuous-time Markov chain with state space \( S = \{1, 2, 3\} \) has generator

\[
G = \begin{pmatrix}
-4 & 2 & 2 \\
1 & -3 & 2 \\
1 & 0 & -1 \\
\end{pmatrix}
\]

(a) Suppose that the is currently in state 2 where it has been for 2 minutes after spending 1.5 minutes in state 1. What is the probability that it stays in state 2 another 2 minutes?

(b) Under the conditions in (a), what is the probability that the next jump is to state 3?

(c) Find the transition matrix \( P \) of the jump chain.

2(a) Birds arrive to rest on a branch of a tree where there is room for a maximum of 4 birds. As long as the birds feel safe they do not leave, but there are occasional incidents that scare them and then all of them leave at once. Suppose that such incidents occur on average every 5 minutes, according to a Poisson process, and that birds arrive individually according to a Poisson process such that you can expect 120 arrivals per hour. Find the generator.

(b) Now instead suppose that birds arrive in flocks according to a Poisson process with rate 1 flock per minute. A flock is equally likely to contain 1, 2, 3, or 4 birds. If there is not room for the entire flock on the branch, they fill it up and the remaining birds leave. Scary incidents occur as in (a). Find the generator.