Stochastic Processes, HW1

Practice problems

1. Book, page 517–518: Problems 1, 3, 4(a),6 7. In all problems, identify the transition matrix and sketch the transition graph. In problem 3, skip the "long-term" part for now.

2. Find the transient states, the recurrent states, and the equivalence classes for the Markov chains with the following transition matrices. Also decide whether each chain is irreducible. The state spaces are of the type $\{1, 2, ..., n\}$. You do not need to formally prove transience/recurrence but make sure you understand intuitively which states are and which are not.

(a)

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(b)

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)

$$P = \begin{pmatrix} 0.3 & 0 & 0.7 \\ 0.2 & 0.8 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(d)

$$P = \begin{pmatrix} 1/3 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 1/4 & 3/4 & 0 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 4/5 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{pmatrix}$$

Turn-in problems, due Wednesday 1/28

1. Book, page 518: Problem 2.

2. True or false: All recurrent states must belong to the same equivalence class. Give a proof or a counterexample.

3(a) Consider a sequence of independent coin flips. Show that the sequence is a Markov chain on the state space $S = \{H, T\}$ (that is, show that the sequence $X_0, X_1, ...$ satisfies the Markov property) and give the transition matrix and transition graph.

(b) Now consider a sequence of independent coin flips where you record overlapping sequences of length 2 (for example, if the sequence is $HHTHT$, we record $X_0 = HH, X_1 = HT, X_2 = TH, X_3 = HT$) and give the transition matrix and transition graph.

(c) If $X_0 = HH$, what is the probability that $X_3 = TT$?

4. Consider a Markov chain with state space $\{0, 1, 2\}$ and the following transition matrix:

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-p & 0 & p \end{pmatrix}$$

where $0 < p < 1$.

(a) Use the definition of recurrence (in terms of the return times τ_i) to show that state 1 is recurrent.
(b) Once we know that state 1 is recurrent, give an intuitive arguments that states 0 and 2 are also recurrent.