Stochastic Processes, HW2

1. Book p.517–518: Find the stationary distributions in problems 1–3.

Turn-in problems

1. Consider the queueing example we did in class where arrivals and departures are equally as likely but suppose that there is only room for 3 customers to wait in line (thus, the maximum number of customers in the system is 4 and if the system is full, arriving customers are turned away). Give the transition graph, the transition matrix, and find the stationary distribution.

2(a) Find the stationary distribution in Problem 4 on HW1.

(b) If you have done **(a)** correctly, π_0 is equal to π_1. Explain why this makes intuitive sense.

3. Consider a Markov chain on some state space S that has two different stationary distributions π and τ. Let α be a real number in $[0, 1]$ and define the vector ν as the convex linear combination $\nu = \alpha\pi + (1 - \alpha)\tau$. Show that ν is a stationary distribution for any such α (remember that there are two properties you need to establish). Thus, a Markov chain has either 0, 1, or (uncountably) infinitely many stationary distributions.