

Stochastic Processes, Spring 2017, Test 2

1. Consider a continuous time Markov chain on the state space $S = \{0, 1, 2\}$ where the jump chain has transition matrix

$$P = \begin{pmatrix} 0 & 1/3 & 2/3 \\ 1/3 & 0 & 2/3 \\ 1 & 0 & 0 \end{pmatrix}$$

and the holding time parameters (rates) in the three states are 9, 6, and 3, respectively.

- (a) Find the generator G .
- (b) Find the stationary distributions ν of the jump chain and π of the continuous time chain.
- (c) In the long run, what percentage of time does the chain spend in state 0?

2. If a service system has so many servers that virtually nobody ever has to wait in line, we can assume that the number of servers is infinite and get a queueing system denoted $M/M/\infty$. Sketch the rate diagram, find the generator, and decide when a stationary distribution exists and what it is expressed in terms of $\rho = \lambda/\mu$.

4. Consider the following variant of the $M/M/1$: A customer who arrives to a system in state k joins with probability $1/(k+1)$ and leaves otherwise, for $k = 0, 1, 2, \dots$. Sketch the rate diagram, find the generator, and decide when a stationary distribution exists and what it is expressed in terms of $\rho = \lambda/\mu$.

5. Consider a bacterial population where individuals have lifetimes that are $\exp(\alpha)$. There is immigration into the population such that groups of immigrants arrive according to a Poisson process with rate λ . A group is of size k with probability p_k for $k \geq 1$. Individual bacteria reproduce by dividing where the probability that a bacterium in a population of size k divides rather than dies equals q_k for $k \geq 1$. As long as $k \leq 10^4$ there is no death, that is, $q_k = 1$ for $k \leq 10^4$. Occasionally, there is a disaster that kills the entire bacterial population. Suppose that disasters occur according to a Poisson process with rate $\mu = 0.1\lambda$. What is the probability that an arriving immigrant finds the population extinct?

6. Write a poem about the class. It should contain rhymes (approximate rhymes are OK) of the following words: stationary, transition, periodic, irreducible, and Persian.