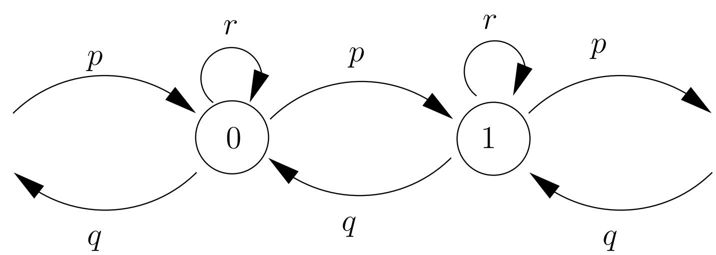
Stochastic Processes, Solutions 4

1(a) The transition graph is



The chain is irreducible if $p > 0$ and $q > 0$. A given state is aperiodic if $r > 0$. Note that if $r = 1$, each state has period 1 but the chain is not irreducible so it does not make much sense to call the entire chain aperiodic. It is recurrent if $p = q$ and $r < 1$ (again, if $r = 1$, the chain is not irreducible and each state is positive recurrent).

(b) Let $x = P_0(\tau_1 < \infty)$ and condition on the first step to get

$$x = p \cdot 1 + r \cdot x + q \cdot x^2$$

which has solutions $x = 1$ and $x = p/q$. As the walk is obviously transient if $p < q$, the answer is $P_0(\tau_1 < \infty) = p/q$

(c) Let $\mu = E_0[\tau_1]$ and condition on the first step to get

$$\mu = p \cdot 1 + r(1 + \mu) + q(1 + 2\mu) = 1 + (r + 2q)\mu$$

which has solution

$$\mu = \frac{1}{1 - r - 2q} = \frac{1}{p - q}$$

Note that $\mu = \infty$ is also a solution. It can be shown this is not the correct one, we skip the argument here.