Stochastic Processes, HW5

Practice problems

1(a) Birds arrive to rest on a branch of a tree where there is room for a maximum of 4 birds. As long as the birds feel safe they do not leave, but there are occasional incidents that scare them and then all of them leave at once. Suppose that such incidents occur on average every 5 minutes, according to a Poisson process, and that birds arrive individually according to a Poisson process such that you can expect 120 arrivals per hour. Find the generator.

(b) Now instead suppose that birds arrive in flocks according to a Poisson process with rate 1 flock per minute. A flock is equally likely to contain 1, 2, 3, or 4 birds. If there is not room for the entire flock on the branch, they fill it up and the remaining birds leave. Scary incidents occur as in **(a)**. Find the generator.

Turn-in problems

1. A continuous-time Markov chain with state space $S = \{1, 2, 3\}$ has generator

$$G = \begin{pmatrix} -4 & 2 & 2 \\ 1 & -3 & 2 \\ 1 & 0 & -1 \end{pmatrix}$$

(a) Suppose that the is currently in state 2 where it has been for 2 minutes after spending 1.5 minutes in state 1. What is the probability that it stays in state 2 another 2 minutes?

(b) Under the conditions in **(a)**, what is the probability that the next jump is to state 3?

(c) Find the transition matrix P of the jump chain.

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3. Consider a queueing system where arrivals occur according to a Poisson process with rate 1 and service times are independent $\sim \exp(1)$. There are 3 servers and no waiting room (customers who arrive when all servers are busy are lost).

(a) Sketch the rate diagram for the continuous-time chain and the transition diagram for the jump chain.

(b) State the generator for the continuous-time chain and the transition matrix for the jump chain.

(c) Find the stationary distribution π for the continuous-time chain and the stationary distribution ν for the jump chain. Verify that ν is proportional to $\lambda \bullet \pi$ (find c such that $\nu_k = c\lambda(k) \cdot \pi_k$ for $k = 0, 1, 2, 3$).

(d) In a given 2-hour period, how many minutes can you expect the system to be empty?

(e) Now add room for one customer to wait in line and suppose that an arriving customer who finds all servers busy stays to wait in line with probability p, and leaves with probability $1 - p$. Sketch the rate diagram for the continuous-time chain and the transition graph for the jump chain.