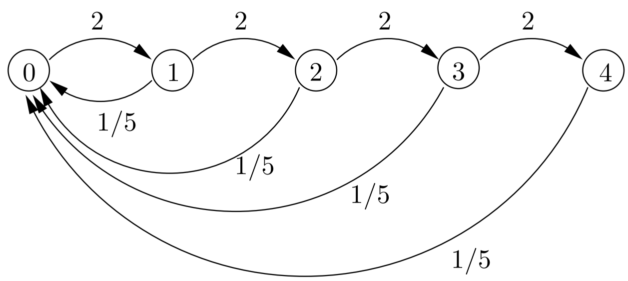
# Stochastic Processes, Solutions 5

## Practice problems

**1(a)** The state space is $S = \{0, 1, 2, 3, 4\}$. Assume that arrivals and incidents occur according to independent Poisson processes with arrival rate 2 and incident rate 1/5 (time unit: minutes). This gives the rate diagram




and the generator

$$G = \begin{pmatrix} -2 & 2 & 0 & 0 & 0 \\ 1/5 & -11/5 & 2 & 0 & 0 \\ 1/5 & 0 & -11/5 & 2 & 0 \\ 1/5 & 0 & 0 & -11/5 & 2 \\ 1/5 & 0 & 0 & 0 & -1/5 \end{pmatrix}$$

**(b)** The arrival rates of flocks of different sizes are thinned Poisson processes with rate 1/4. We get the generator

$$G = \begin{pmatrix} -1 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/5 & -6/5 & 1/4 & 1/4 & 1/2 \\ 1/5 & 0 & -6/5 & 1/4 & 3/4 \\ 1/5 & 0 & 0 & -6/5 & 1 \\ 1/5 & 0 & 0 & 0 & -1/5 \end{pmatrix}$$

## Turn-in problems

**1(a)** By the Markov property, the remaining time $T$ is $\exp(\lambda(2))$ where $\lambda(2) = -\gamma_{22} = 3$ and we get $P(T > 2) = e^{-3 \cdot 2} = e^{-6} \approx 0.0025$.

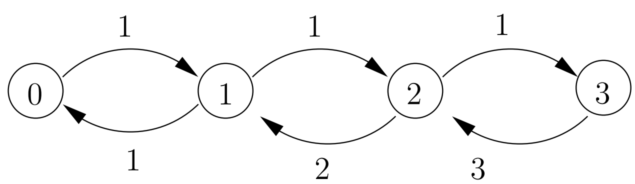
**(b)** This probability is $p_{23} = -\gamma_{23}/\gamma_{22} = 2/3$.

**(c)** We have $p_{ii} = 0$ and $p_{ij} = -\gamma_{ij}/\gamma_{ii}$ for $i \neq j$ which gives

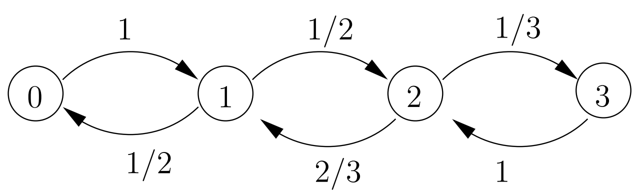
$$P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 0 & 2/3 \\ 1 & 0 & 0 \end{pmatrix}$$

**2.** Identically 0. As the rate is 0 to go to any other state, the off-diagonal entries are all equal to 0 and as the row sum must equal 0, the diagonal entry is 0 as well.

**3(a)** The rate diagram is




and the transition diagram is




**(b)** The generator is

$$G = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & 2 & -2 \end{pmatrix}$$

and the transition matrix is

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

**(c)** Solving $\pi G = 0$ gives

$$\pi = \left(\frac{6}{16}, \frac{6}{16}, \frac{3}{16}, \frac{1}{16}\right)$$

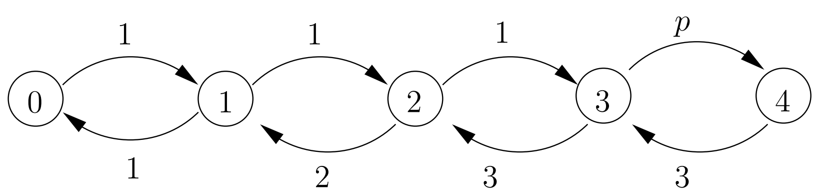
Solving $\nu P = \nu$ gives

$$\nu = \left(\frac{2}{10}, \frac{4}{10}, \frac{3}{10}, \frac{1}{10}\right)$$

and it is easily verified that with $c = 8/15$, we have $\nu_k = c\lambda(k)\pi_k$ for $k = 0, 1, 2, 3$.

**(d)** The long-term proportion the system is empty is $\pi_0 = 6/16$ and since $2 \cdot 6/16 = 3/4$, we expect the system to be empty for a total of 45 mintues.

**(e)** Transitions from 3 to 4 are described by a thinned Poisson process with rate $\lambda p$ so the rate diagram is




which gives the transition diagram