

Stochastic Processes, Final Homesork, Spring 2017

1. Book p. 259: 137.

2. Consider a cell population where cells die with probability $1/3$ and divide with probability $2/3$. **(a)** Find the extinction probability of a population started from a single ancestor. **(b)** If you can choose the number of ancestors and want the extinction probability to be at most 1%, how many ancestors should you have?

3. Consider a branching process where the offspring distribution is a binomial distribution with $n = 10$ and $p = 0.1$. Find **(a)** $G(s)$ **(b)** $G_2(0) = P(Z_2 = 0)$ **(c)** $P(E)$

4. Find the extinction probabilities for the following branching processes.

(a) $P(X = k) = \frac{90}{\pi^4 k^4}, \quad k = 1, 2, \dots$

(b) The offspring distribution is the so-called truncated geometric distribution which has

$$P(X = 0) = p_0$$

$$P(X = k) = cp^k, \quad k = 1, 2, \dots$$

where p_0, p , and c must satisfy

$$p_0 + c \frac{p}{1-p} = 1$$

and it can be shown that the mean is

$$E[X] = \frac{cp}{(1-p)^2}$$

Let $p_0 = 2/3$ and $p = 1/2$.

(c) The offspring distribution has pgf $G(s) = e^{-0.5+0.5s}$

(d) An individual is equally as likely to have 0 or 3 offspring. Comment on the solution!