Math 1312 Fall 2010

Calculus II

Second Midterm Exam

Monday, October 25

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators or other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ______________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**
Points	8	22	10	25	15	10
Score						

Total:____________________

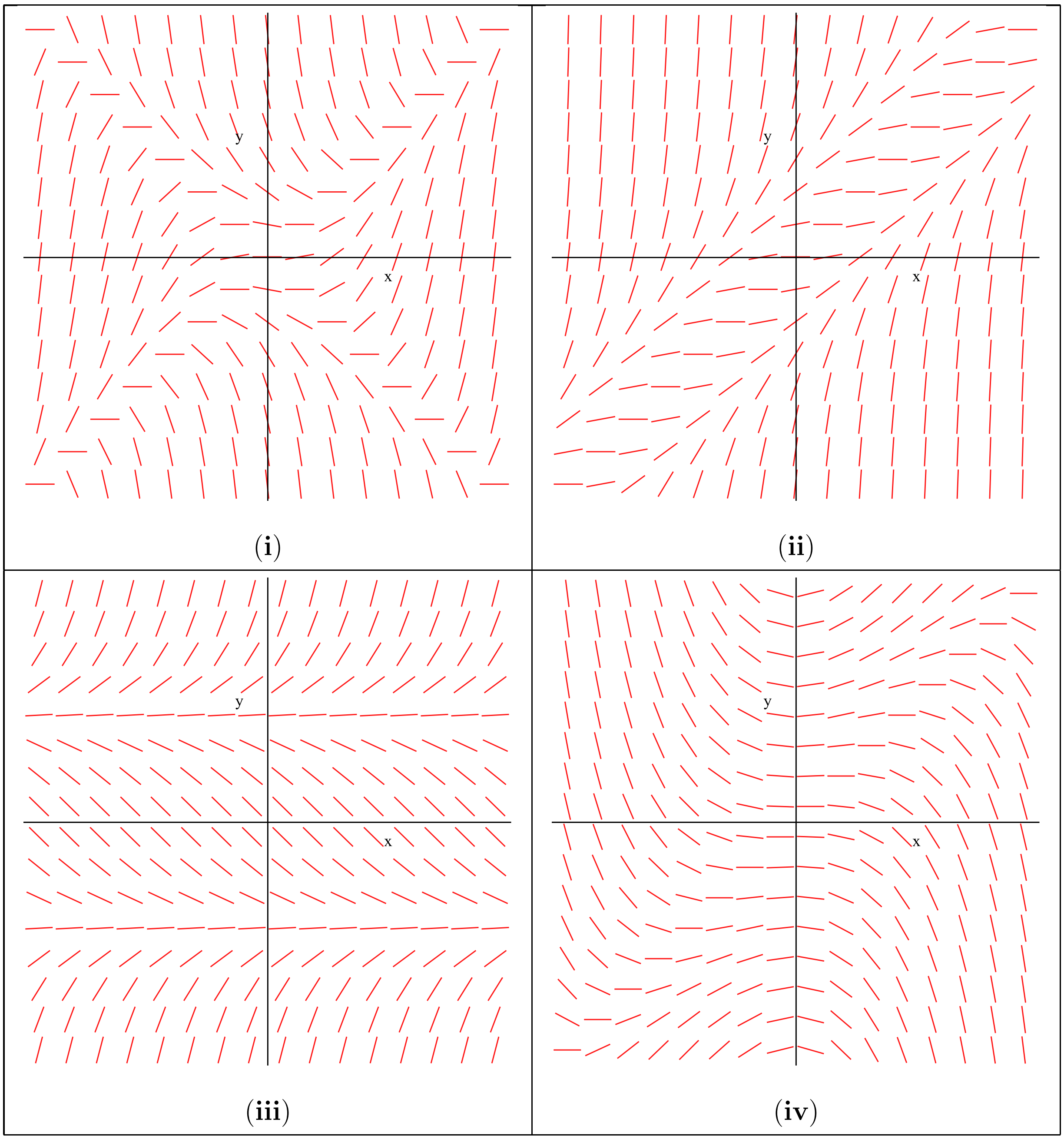
1. Match the following differential equations (**a** - **d**) with their slope fields (**i** - **iv**). You do not need to justify your answers.

_____ **a.** $\dfrac{dy}{dx} = x^2 - y^2$

_____ **b.** $\dfrac{dy}{dx} = (x - y)^2$

_____ **c.** $\dfrac{dy}{dx} = y^2 - 1$

_____ **d.** $\dfrac{dy}{dx} = xy - x^2$



2. Solve the initial value problem.

a. $t\dfrac{dw}{dt} + 2w = t^3$, $w(1) = 0$

b. $xy' + y = y^2$, $y(1) = -1$

3. Determine the values of a and b so that $y = e^{x/2}\left(c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)\right)$ is the general solution to $4y'' + ay' + by = 0$.

4. The functions $y_1 = e^{2x}$ and $y_2 = e^{-3x}$ are both solutions to the homogeneous linear differential equation $y'' + y' - 6y = 0$.

a. Find a solution to the inhomogeneous linear differential equation $y'' + y' - 6y = 1 + x$.

b. Find a solution to the inhomogeneous linear differential equation $y'' + y' - 6y = e^{2x}$.

c. Use parts **a** and **b** to find the general solution to the inhomogeneous linear differential equation $y'' + y' - 6y = 1 + x + e^{2x}$.

5. A 4 kg mass is suspended by a spring. It requires 5 N of force to stretch and hold the spring 20 cm beyond its equilibrium position.

a. Determine the spring constant k.

b. Determine the value of the damping constant c that will critically damp the spring-mass system.

c. If we subject the spring-mass system to critical damping, compress the spring 50 cm above equilibrium, and release it with a downward velocity of 0.5 m/s, will the mass ever cross through its equilibrium position? If so, at what time?

6. Consider the differential equation

$$x^2y'' - 8xy' + 20y = 0. \tag{1}$$

Notice that this equation is linear, but that it does not have constant coefficients.

a. Determine the two values of r for which $y = x^r$ is a solution to (1).

b. Use part **a** to find the solution to (1) that satisfies $y(-1) = 0$, $y'(-1) = 3$.