MATH 1312 FALL 2008

CALCULUS II

FINAL EXAM

MONDAY, DECEMBER 15, 6:30 PM - 9:30 PM

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	1	2	3	4	5	6	7	8	9
Points	15	10	6	8	10	10	9	16	9
Score									

Problem	10	11	12	13	14	15
Points	9	5	10	8	15	10
Score						

Total: _____

1. Let $\mathbf{a} = \langle -1, 3, 2 \rangle$ and $\mathbf{b} = \langle 12, 2, 3 \rangle$.

  a. Compute $3\mathbf{a} - 2\mathbf{b}$.

  b. Compute $|\mathbf{a} + \mathbf{b}|$.

  c. Compute $\mathbf{b} \times \mathbf{a}$.

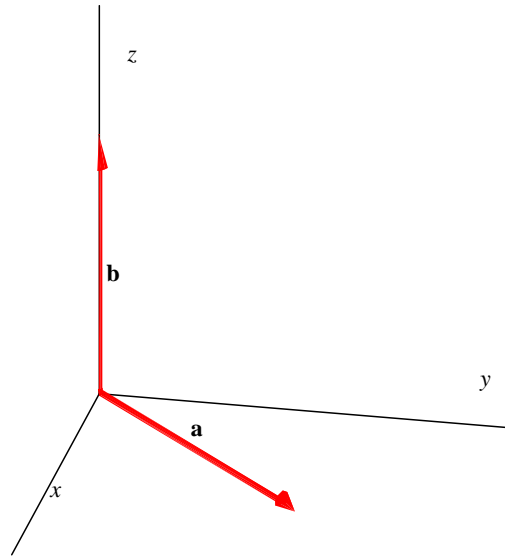 **d.** Are $\mathbf{a}$ and $\mathbf{b}$ parallel, perpendicular, or neither?

  e. Find a vector of length 5 that points in the same direction as $\mathbf{a}$.

2. Consider the three points $P(2, 1, 5)$, $Q(-1, 3, 4)$ and $R(3, 0, 6)$. Let T denote the triangle with vertices P, Q, and R.

    a. Find the area of T.

    b. Is T a right triangle?

3. The figure shows a vector **a** in the xy-plane and a vector **b** in the direction of **k**. Assume that $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$.

a. Find $\mathbf{a} \cdot \mathbf{b}$.

b. Find $|\mathbf{a} \times \mathbf{b}|$.

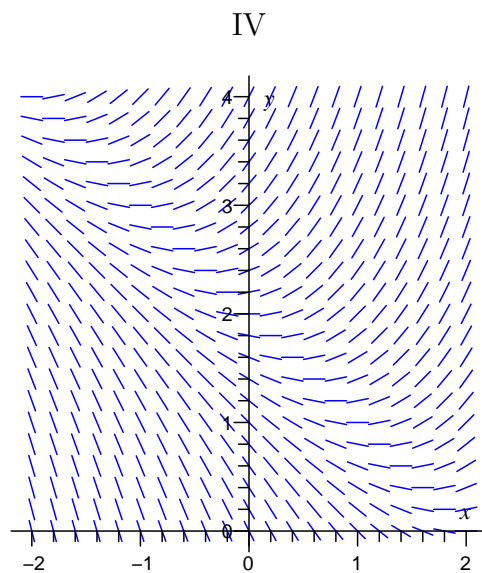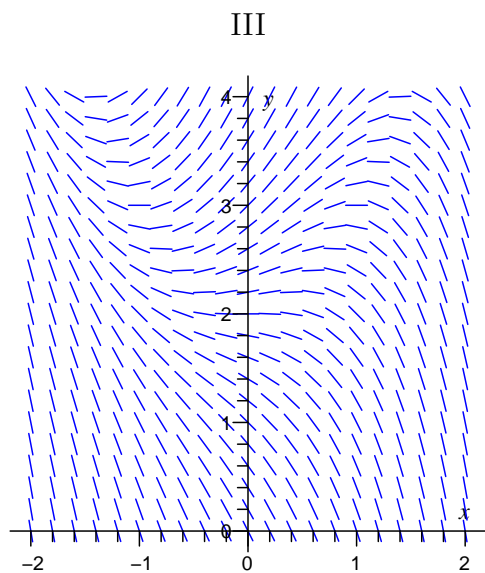**c.** Determine if the components of $\mathbf{a} \times \mathbf{b}$ are positive, negative or zero.
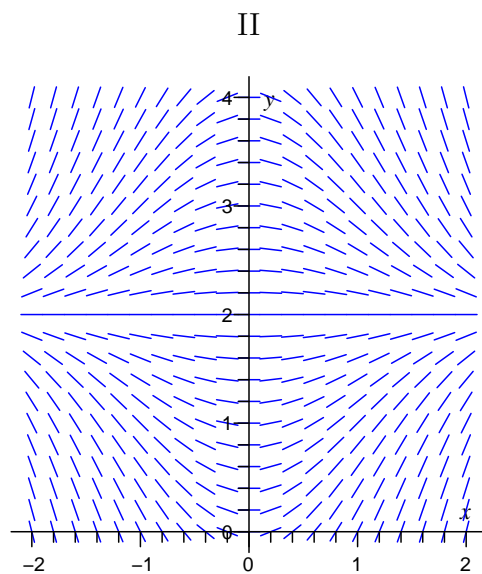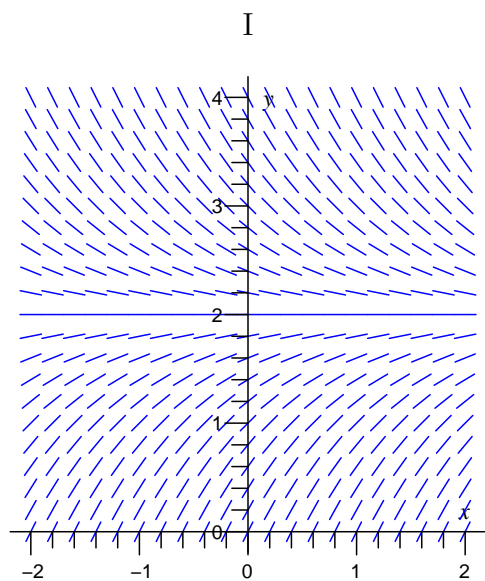
4. Match the differential equation with its direction field.

a. $y' = y - x^2 - 2$ _____

b. $y' = 2 - y$ _____

c. $y' = x(2 - y)$ _____

d. $y' = y + x - 2$ _____

I

II

III

IV

5. Determine all values of x for which the power series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n3^n}$ converges.

6.

a. Find the angle between the vectors $\mathbf{u} = \langle 1, 0, 1 \rangle$ and $\mathbf{v} = \langle 0, -1, 1 \rangle$.

b. If $A = \begin{pmatrix} 5 & 2 & 0 \\ -3 & 0 & 1 \end{pmatrix}$, find a nonzero vector $\mathbf{v}$ so that $A\mathbf{v} = \mathbf{0}$.

7. Find a number r so that $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \ln r.$

8. Let

$$A = \begin{pmatrix} 2 & -5 \\ 0 & -2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 3 \end{pmatrix}, C = \begin{pmatrix} -2 & 0 & 1 \\ 4 & 0 & -1 \\ 3 & 1 & 2 \end{pmatrix}, D = \begin{pmatrix} 0 & -3 \\ 2 & 1 \\ -1 & 5 \end{pmatrix}.$$

Compute the following expressions or explain why they are not defined.

   a. $BA - AB$

   b. $C - D$

c. $AB - BC$

d. $2A + BD$

9. Evaluate $\displaystyle\int_{3\sqrt{2}}^{6} \frac{\sqrt{x^2 - 9}}{x^3}\, dx.$

10. Evaluate $\displaystyle\int_0^2 z^2 \ln z \, dz$.

11. Find a, b and c so that the general solution to $ay'' + by' + c = 0$ is $y = c_1 e^{-x} + c_2 e^{3x}$.

12. Solve the initial value problem $\dfrac{dy}{dt} = \dfrac{te^t}{y\sqrt{1+y^2}}$, $y(0) = 2\sqrt{2}$.

13. Let R be the region that is bounded by the graph of $y = x^2 - x^3$ and the x-axis. Determine the volume of the solid obtained by rotating R about the line $x = 3$.

14. Determine if the series converges conditionally, converges absolutely, or diverges.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}-1}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$

15. Do the four points $A(1, 1, -1)$, $B(0, -2, 6)$, $C(-2, 3, -13)$ and $D(2, 0, 4)$ all lie on the same plane?