# Math 1312 Fall 2010

## Calculus II

## Final Exam

## Wednesday, December 15

Your name (please print):

**Instructions:** This is a closed book, closed notes exam. **Use of calculators or other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

**The Honor Code requires that you neither give nor receive any aid on this exam.**

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

**Pledged:** ______________________________

Do not write below this line

| Problem | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Points | 36 | 10 | 27 | 5 | 10 | 9 | 12 | 15 | 10 | 8 | 10 |
| Score | | | | | | | | | | | |

**Total:**____________________

**1.** Evaluate the following integrals.

**a.** $\displaystyle\int (\ln z)^2 \, dz$

**b.** $\displaystyle\int_{3/2}^{3} \sqrt{9 - x^2} \, dx$

**c.** $\displaystyle\int \frac{t^3 - 4t - 10}{t^2 - t - 6}\, dt$

**d.** $\displaystyle\int_e^\infty \frac{ds}{s(\ln s)^3}$

**2.** Determine the value of the constant $M$ so that

$$\int_0^1 \frac{1}{x}\left(\frac{1}{x^2+1} - \frac{M}{x+2}\right)\, dx$$

converges, and evaluate the integral for this value of $M$.

**3.** Solve the initial value problem.

**a.** $xy' - y = x \ln x$, $y(1) = 2$

**b.** $y' = 3x^2 e^y$, $y(0) = 1$

**c.** $2y'' + 5y' + 3y = 0$, $y(0) = 3$, $y'(0) = -4$

**4.** A sequence $\{a_n\}$ is defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$. Assuming that it exists, find $\lim_{n\to\infty} a_n$.

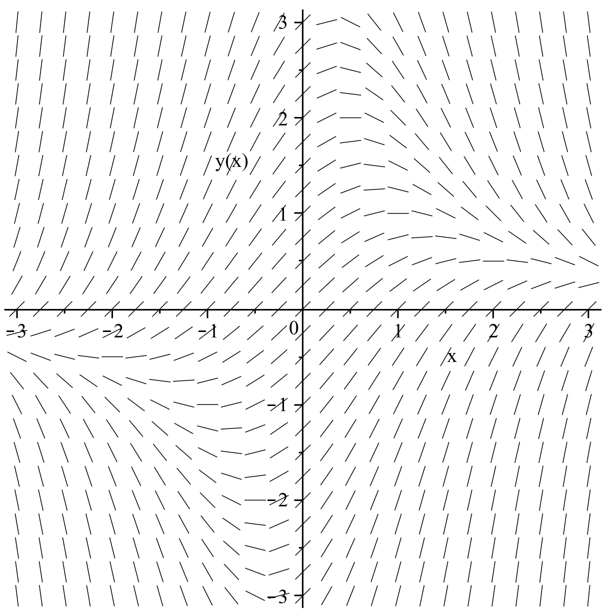
**5.**

**a.** Show that $y = \tan x$ is a solution to the differential equation $y'' - 2y = 2\tan^3 x$.

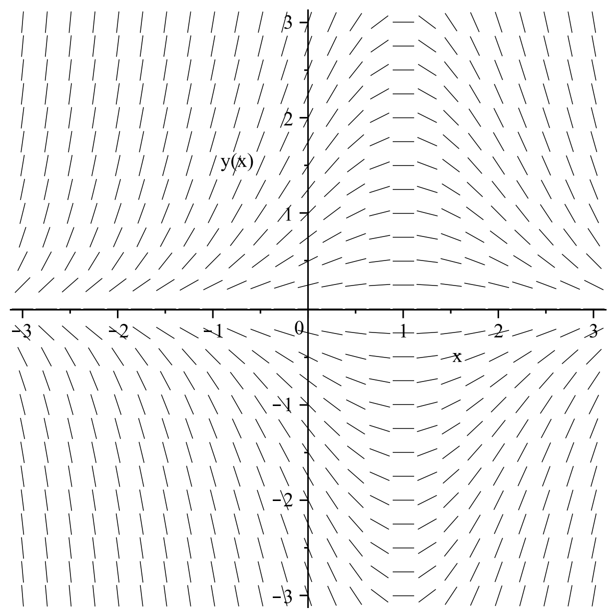
**b.** Use part **a** to find the general solution to $y'' - 2y = 2\tan^3 x$.

**6.** Match each differential equation with its direction field.

_____ **a.** $\dfrac{dy}{dx} = 1 - xy$ _____ **b.** $\dfrac{dy}{dx} = y + xy$ _____ **c.** $\dfrac{dy}{dx} = y - xy$




(i)

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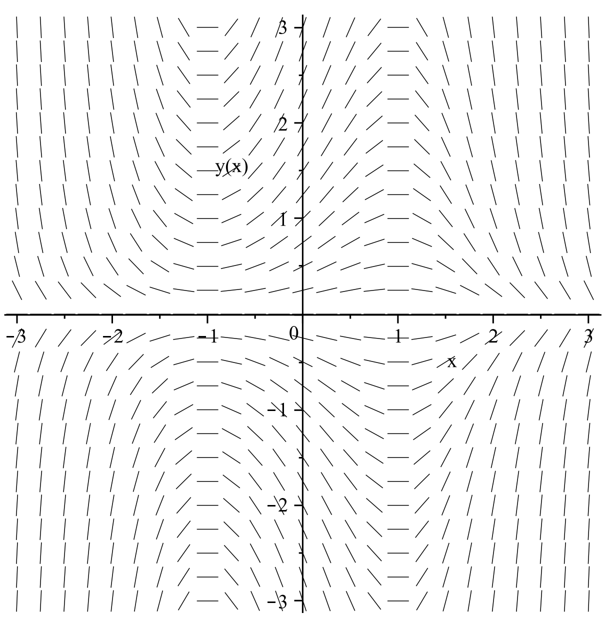


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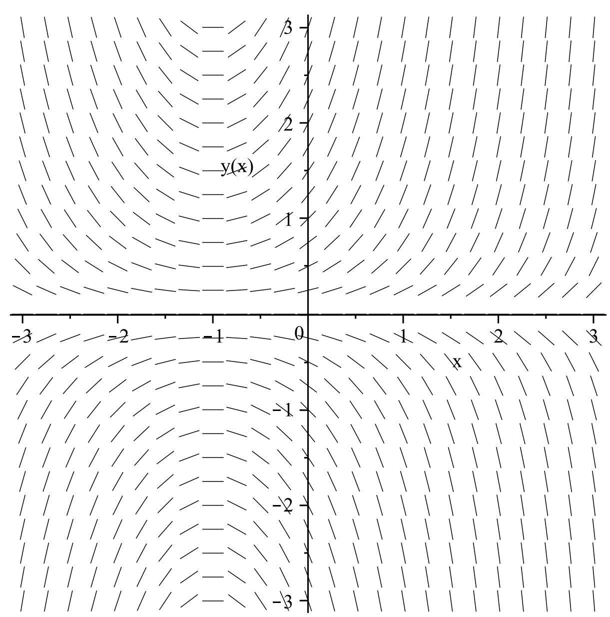


(iv)

**7.** Determine if the following series are absolutely convergent, conditionally convergent, or divergent, and indicate which test(s) you used to arrive at your conclusion. If you use one of the comparison tests, be sure to write down the series to which you are making a comparison. Other than this, you do not need to show any work. An example is shown below.

**Ex.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+2}$ is: *absolutely convergent*

by the: *comparison test, with* $\sum_{n=1}^{\infty} \frac{1}{n^3}$ .

**a.** $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\ln n}{n}$ is: ______________________

by the: ______________________

______________________.

**b.** $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$ is: ______________________

by the: ______________________

______________________.

**c.** $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+1}{5^n}$ is: ______________________

by the: ______________________

______________________.

**d.** $\displaystyle\sum_{n=0}^{\infty} \frac{n!\sin 2n}{(n+2)!}$ is: ______________________

by the: ______________________

______________________.

**8.** Write the following series using summation notation. Find the sum of each series, or show that it does not converge.

**a.** $1 + 0.3 + 0.09 + 0.027 + 0.0081 + \cdots$

**b.** $1 - 2 + 3 - 4 + 5 - 6 + \cdots$

**c.** $1 + \dfrac{1}{2} + \dfrac{1}{2^2 2!} + \dfrac{1}{2^3 3!} + \dfrac{1}{2^4 4!} + \cdots$

**d.** $\pi - \dfrac{\pi^3}{3!} + \dfrac{\pi^5}{5!} - \dfrac{\pi^7}{7!} + \cdots$

**9.** Find the first four terms of the Taylor series for $f(x) = \sqrt{2x+2}$ centered at $a = 1$.

**10.** Find the domain of the function

$$f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^n}{n^2 5^n}.$$

**11.** Let

$$A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 4 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & -1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 3 \\ -2 & 3 \end{pmatrix}.$$

Compute the following quantities, or explain why they do not make sense.

**a.** $A - BC$

**b.** $CB$

**c.** $DC + 2A$

**d.** $DA - AD$