CALCULUS II
SPRING 2011

ASSIGNMENT 4.1
DUE SEPTEMBER 19

You may find the following equation useful in the problems below.

Reduction Formula. For $n \geq 2$

$$\int \frac{du}{(u^2+a^2)^n} = \frac{u}{(2n-2)a^2(u^2+a^2)^{n-1}} + \frac{2n-3}{a^2(2n-2)} \int \frac{du}{(u^2+a^2)^{n-1}}.$$

1-5 Evaluate the integral.

Exercise 1. $\displaystyle\int_0^1 \frac{x}{x^2+6x+10}\,dx$

Exercise 2. $\displaystyle\int \frac{x^2-3x+7}{(x^2-4x+6)^2}\,dx$

Exercise 3. $\displaystyle\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)^2}\,dx$

Exercise 4. $\displaystyle\int \ln(x^2-x+2)\,dx$ [*Suggestion:* First integrate by parts.]

Exercise 5. $\displaystyle\int \frac{du}{(u^2+16)^4}$

Exercise 6. Starting from

$$\int \frac{dx}{(x^2+1)^n} = \frac{x}{(x^2+1)^n} + 2n \int \frac{x^2}{(x^2+1)^{n+1}}\,dx. \tag{1}$$

(which we derived previously), deduce that

$$\int \frac{dx}{(x^2+1)^{n+1}} = \frac{x}{2n(x^2+1)^n} + \frac{2n-1}{2n} \int \frac{dx}{(x^2+1)^n}.$$

This gives the reduction formula in the special case that $a = 1$. [*Suggestion:* Use the identity $x^2 = (x^2+1) - 1$ in the numerator of the integral on the right side of (1).]