Complex Variables
Spring 2011

Assignment 11.2
Due November 16

Exercise 1. Consider the function $f(z) = e^{1/z} - 1$. We have seen that it has a sequence of distinct zeros $\{z_n\}_{n=1}^{\infty}$ with the property that $z_n \to 0$, and yet it is not the zero function. Why doesn't this contradict the theorem on the isolation of zeros of analytic functions?

Exercise 2. Suppose that f has either a zero or a pole of order $n \geq 1$ at z_0. Prove that f'/f has a simple pole at z_0.

Exercise 3. Find all the singularities of

$$f(z) = \frac{(z-2)^2(z+4)}{1-\cos(\pi z)}$$

and identify each as removable, a pole of a certain order, or an essential singularity.

Exercise 4. Repeat the preceding exercise for

$$f(z) = \frac{(z-2i)^4}{\sin(\pi i/z) - 1}.$$