Complex Variables
Spring 2011

Assignment 3.2
Due September 19

Exercise 1. Recall that a function $f(z)$ is *defined on a neighborhood of* ∞ if there exists an $R > 0$ so that the domain of f contains $\{z : |z| > R\}$.

- **a.** Show that if $f(z)$ is defined on a neighborhood of ∞ if and only if $g(w) = f(1/w)$ is defined on a deleted neighborhood of 0.
- **b.** Show that $\lim_{z\to\infty} f(z) = a$ if and only if $\lim_{w\to 0} f(1/w) = a$.

Exercise 2. If $p(z)$ is a non-constant polynomial, prove that $\frac{1}{p(z)}$ is defined on a neighborhood of ∞ and that $\lim_{z\to\infty} \frac{1}{p(z)} = 0$. [*Suggestion:* You can avoid an argument involving ϵ by using the preceding exercise and the limit laws.]

Exercise 3. Let $\log w$ denote the branch of the logarithm with $\arg w \in (-\pi, \pi]$.

- **a.** Where is $\log(z^2)$ continuous? What about $\log(z^3)$?
- **b.** Let $w^{1/2} = e^{\frac{1}{2}\log w}$. Where is $(1 + z)^{1/2}$ continuous? What about $(1 - z)^{1/2}$?