Complex Variables
Spring 2011

Assignment 9.1
Due October 31

Exercise 1. Use Cauchy's Integral Formula to evaluate the following definite integrals. All contours are positively oriented.

a. $\displaystyle\int_\gamma \frac{\sin e^z}{z}\,dz$ where γ is any simple closed curve containing the origin.

b. $\displaystyle\int_\gamma \frac{z}{(z-a)(z-b)}\,dz$ where γ is any simple closed curve containing the distinct complex numbers a and b.

c. $\displaystyle\int_\gamma \frac{e^{z^2}}{z^3-1}\,dz$ where γ is the circle $|z-1|=1$.

Exercise 2. Show that if $a>1$ then

$$\int_0^{2\pi} \frac{d\theta}{a+\cos\theta} = \frac{2\pi}{\sqrt{a^2-1}}.$$

[*Hint:* You can realize this integral as the parametrized form of an integral over the unit circle by setting $z=e^{i\theta}$.]

Exercise 3. Suppose that f is analytic on a disk D and that γ is a simple closed curve in D. Suppose that f is constant on γ. Prove that f takes on the same constant value everywhere inside γ as well.