



PUTNAM EXAM SEMINAR  
FALL 2012

ASSIGNMENT 11  
DUE NOVEMBER 12

**Exercise 1.** Consider a set  $S$  with a binary operation  $*$ , that is, for each  $a, b \in S$ ,  $a * b \in S$ . Assume that  $(a * b) * a = b$  for all  $a, b \in S$ . Prove that  $a * (b * a) = b$  for all  $a, b \in S$ . [Putnam 2001, A1].

**Exercise 2.** Let  $S$  be a non-empty set with an associative operation that is left and right cancellative ( $xy = xz$  implies  $y = z$  and  $yx = zx$  implies  $y = z$ ). Assume that for every  $a$  in  $S$  the set  $\{a^n : n = 1, 2, 3, \dots\}$  is finite. Must  $S$  be a group? [Putnam 1989, B2]