Exercise 1. Suppose $a_1 = 1$, $a_2 = 5$ and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 3$. Determine a_n.

Exercise 2. Suppose that $a_0 = a_1 = 1$ and $a_n = 6a_{n-1} - 9a_{n-2} - 2^n$ for $n \geq 2$. Find a_{2008}.

Exercise 3. Let $x_1, x_2, x_3, \ldots$ be a sequence of nonzero real numbers satisfying

$$x_n = \frac{x_{n-2}x_{n-1}}{2x_{n-2} - x_{n-1}} \text{ for } n \geq 3.$$

Establish necessary and sufficient conditions on x_1 and x_2 for x_n to be an integer for infinitely many values of n.

Exercise 4. The sequence of integers $u_0, u_1, u_2, \ldots$ satisfies $u_0 = 1$ and

$$u_{n+1}u_{n-1} = ku_n \text{ for all } n \geq 1,$$

where k is some fixed positive integer. If $u_{2012} = 2012$, what are the possible values of k?

Exercise 5. Let $\{x_n\}$ be a sequence of nonzero real numbers such that $x_n^2 - x_{n-1}x_{n+1} = 1$ for $n \geq 1$. Prove that there is a real number a so that $x_{n+1} = ax_n - x_{n-1}$ for all $n \geq 1$. [Putnam Exam, 1993, A-2]