Exercise 1. Find the unique function $u(t)$ so that

$$u'(t) = u(t) + \int_0^1 u(s)\,ds$$

and $u(0) = 1$. [Putnam 1958, 3]

Exercise 2. Find every real-valued function f whose domain is an interval I having 0 as its left-hand end point, such that for every positive x in I, the average value of f on the interval $[0, x]$ is equal to the geometric mean of $f(0)$ and $f(x)$.