



PUTNAM EXAM SEMINAR  
FALL 2012

ASSIGNMENT 8  
DUE OCTOBER 24

**Exercise 1.** Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots? [Putnam 2002, B1]

**Exercise 2.** Prove that it is impossible to load a pair of dice so that every sum  $2, 3, \dots, 12$  is equally likely. Assume that the dice are distinguishable, e.g. a 2 on the first die and a 4 on the second die is different from a 4 on the first die and a 2 on the second, even though the same sum of 6 is obtained.

**Exercise 3.** You have coins  $C_1, C_2, \dots, C_n$ . For each  $k$ ,  $C_k$  is biased so that, when tossed, it has probability  $1/(2k + 1)$  of landing heads up. If the  $n$  coins are tossed, what is the probability that the number of heads is odd? Express your answer as a rational function of  $n$ . [Putnam 2001, A2]