Putnam Exam Seminar
Fall 2012

Quiz 1
August 29

Problem 1. A function f is defined for all positive integers and satisfies

$$f(1) = 2012$$

and

$$f(1) + f(2) + \cdots + f(n) = n^2 f(n).$$

Compute the exact value of $f(2012)$.

Problem 2. Let $n \geq 1$. Prove that $2^{2^n} - 1$ has at least n distinct prime factors.

Problem 3. Show that every positive integer can be written as the sum of integers of the form $2^s 3^t$, such that no summand divides another.