

Putnam Exam Seminar Fall 2012 Quiz 2 September 12

**Problem 1.** Given any thirteen real numbers, prove that there are two of them, say x and y, so that

0 < x - y	.	$2-\sqrt{3}$
$0 < \frac{1}{1+xy} < \sqrt{1+xy}$	V	$\overline{2+\sqrt{3}}$ .

**Problem 2.** Given a set of n + 1 positive integers, none of which is greater than 2n, prove that at least two of the integers are relatively prime.

**Problem 3.** A college senior is studying for the MCAT, which is 11 weeks away. She decides to study at least an hour a day, but no more than 12 hours in any week. Show that there is a sequence of consecutive days in which the senior studies exactly 20 hours.